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**A GENERAL SURVEY OF CLASSICAL CEPHEID VARIABLE STARS**

By Robert M. Wilson  
Space Sciences Laboratory

September 16, 1969



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*George C. Marshall Space Flight Center  
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Robert M. Wilson\*

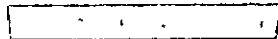
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### ABSTRACT

This report concerns Cepheid Variable Stars, their description, their characteristics and their spectra. Because of the importance of this subject, other areas outside of the immediate topical discussion are reviewed. These include the interstellar medium, infrared radiation (history, detection, atmospheric effects), and the H-R Diagram. The topics of immediate concern which are discussed are Cepheids, types I and II, the light curve, the velocity curve, pulsation theory (without mathematical proof), distance modulation, and needs for research. Direct quotations are provided to express the idea more effectively, and special references are used for completeness.

\* Student trainee from University of South Florida, Tampa, Florida.

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A GENERAL SURVEY OF CLASSICAL  
CEPHEID VARIABLE STARS

By

Robert M. Wilson

SPACE SCIENCES LABORATORY  
RESEARCH AND DEVELOPMENT OPERATIONS

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# A GENERAL SURVEY OF CLASSICAL CEPHEID VARIABLE STARS

## SUMMARY

The following is a report on cepheid variable stars. Many topics concerning the subject are discussed, and further references are provided at the end of the text for the individual who desires additional details.

The first two sections are concerned with the description and distinction of cepheid variable stars and provide a general introduction for the text. Such topics as intrinsic variable stars, tables of pulsating variables, criteria for classification, frequency-variable occurrence tables, spectral types, and type I and II cepheids are discussed.

The sections entitled The Light Curve, The Velocity Curve, and Other Relationships, discuss techniques used in measuring light intensity, the elements of a light curve, the Hertzsprung Relation, definitions of a light and velocity curve, methods to calculate  $\Delta R$ , Wesselink's method for calculating  $\bar{R}$ , the period-spectrum relation, the period-luminosity relation, the spectral-type-intrinsic color relation, the period-amplitude relation, comparison of the major curves and relationships, and the period-luminosity curve in the H-R diagram.

The section entitled Spectra and Radiation Curves discusses Planck's radiation law and Wien's law. A description of cepheid spectra is given with some discussion concerning pulsation theory and evolutionary aspects of cepheids. The section concludes with a discussion of six-color photometry of variable stars (Polaris,  $\delta$  Cephei and  $\eta$  Aquilae).

The interstellar medium and distance modulation are discussed in the next section. Such topics as extinction, color excess, polarization, and intrinsic color are also discussed.

The last two sections contain a discussion of astronomical infrared measurements and cepheid research along these lines. Such topics as detectors, atmospheric windows, absorption, earth-bound and satellite research are discussed.



## INTRODUCTION

There are many stars in the universe which seem to vary in intensity. Many of these are eclipsing binaries; others are explained as novae, and still others are pulsating variables.

Pulsating, or intrinsic variable stars fluctuate or vary in brightness because of inherent forces within the stars themselves, but cannot be considered eclipsing binaries. The first such pulsating star was discovered by D. Fabricius in 1596. This star was Mira Ceti, or  $\alpha$  Ceti. The first classical cepheid variable was discovered in 1784 by a young English amateur astronomer, John Goodricke, who discovered the variation just two years before his death at the age of 21.

Table I [1] summarizes pulsating variable characteristics.

Classical cepheids are very luminous supergiant stars; their absolute visual magnitudes range from  $-1.5$  to about  $-5$ . They are yellow stars and belong to the spectral classes F and G. Generally speaking, the greater the average luminosity of a cepheid the larger its color index and the later its spectral type. The most common period among the classical cepheids appears to be seven days and the typical amplitude about one magnitude. The light curve is generally asymmetrical, i. e., the rising branch is steeper than the declining branch. Normally, the time of rise comprises about one-third of the total period. A relatively short time is spent at maximum light, while the decline in brightness proceeds slowly and smoothly to minimum light, which is usually longer and flatter than the maximum. The prototype of the classical cepheids is  $\delta$  Cephei with a period 5.37 days. The prototype of the type II cepheids is W Virginis with a period of 17.27 days.

The standard international reference of stars that shows variation in light is the Soviet General Catalogue of Variable Stars (1958 - 2nd edition), which lists over 14 000 known variable stars in our galaxy.

In the study of variable stars, a knowledge of rational criteria for classification purposes will be needed. Research [2] concerning this resulted in the following choice:

1. The light variation, its regularity, the form of the light curve, its period, and its amplitude.

TABLE I. PULSATING VARIABLES

Type of variable	Spectra	Period (days)	Median magnitude (absolute)	Amplitude (magnitudes)	Description	Example	Number known in Galaxy*
Cepheids (type I)	F to G supergiants	3 to 50	-1.5 to -5	0.1 to 2	Regular pulsation; period-luminosity relation exists	$\delta$ Cep	610
Cepheids (type II)	F to G supergiants	5 to 30	0 to -3.5	0.1 to 2	Regular pulsation; period-luminosity relation exists	W Vir	(About 50; included with type I)
RV Tauri	G to K yellow and red bright giants	30 to 150	-2 to -3	Up to 3	Alternate large and small maxima	RV Tau	92
Long-period (Mira type)	M red giants	80 to 1000	+2 to -2	>2.5	Brighten more or less periodically	$\alpha$ Cet (Mira)	3657
Semiregular	Mostly M giants	30 to 1000	0 to -3	1 to 2	Periodicity not dependable; often interrupted	Z Ari	1675
Irregular	All types	Irregular	<0	Up to several magnitudes	No known periodicity; many may be semiregular, but too little data exist to classify them as such	CQ Cyg	1370
RR Lyrae or cluster	A to F blue giants	<1	0 to +1	<1 to 2	Very regular pulsations	RR Lyr	2426
$\beta$ Cephei or $\beta$ Canis Majoris	B blue giants	0.1 to 0.3	-2 to -4	0.1	Maximum light occurs at time of highest compression	$\beta$ Cep	11
$\delta$ Scuti	F subgiants	<1	0 to +2	<0.25	Similar to, and possibly related to RR Lyrae variables	$\delta$ Sct	5
Spectrum variables	A main-sequence	1 to 25	0 to +1	0.1	Anomalously intense lines of Si, Sr, and Cr vary in intensity with same period as light; most have strong variable magnetic fields	$\alpha^2$ C Vn	9

\*According to 1958 Edition of Soviet General Catalogue of Variable Stars

2. The spectrum, its peculiarities relative to normal (non-variable) stars, the presence and kind of emission lines, the variations of intensity and wavelength (Doppler effect) of spectral lines, and their profiles.

3. The position in the Hertzsprung-Russell diagram.

4. The type of stellar population or, more generally, the statistical properties in the galactic system (space and velocity distribution).

5. The theoretical interpretation.

The most typical characteristic of cepheids is their amazing regularity in all respects. Two cepheids of the same period and stellar population are, apparently, exact replicas of each other.

The following conclusions [3] may be derived from an inspection of the groups of pulsating stars:

1. Pulsating variables appear in all spectral classes, B to M.

2. With the exception of the dwarf cepheids, all pulsating stars are giants or supergiants.

3. The periods of all variables of any one group increase with decreasing density of stars.

4. Different groups seem to be associated with the spiral arms (population I) and the halo (population II) population of galaxies.

"Among the 254 stars known to be within a distance of 30 light years from the Sun, we find no supergiants and only one normal giant (Pollux in Gemini), but no fewer than seven white dwarfs" [4]. No cepheids are observed in this limited range.

Because the cepheids are found near the plane of the galaxy (normally, within 250 parsec of the plane of the Milky Way), they sometimes appear heavily reddened, and corrections must be applied in their distance moduli for accurate galactic distance determinations.

The frequency of variable occurrence is shown in Figure 1 [5].

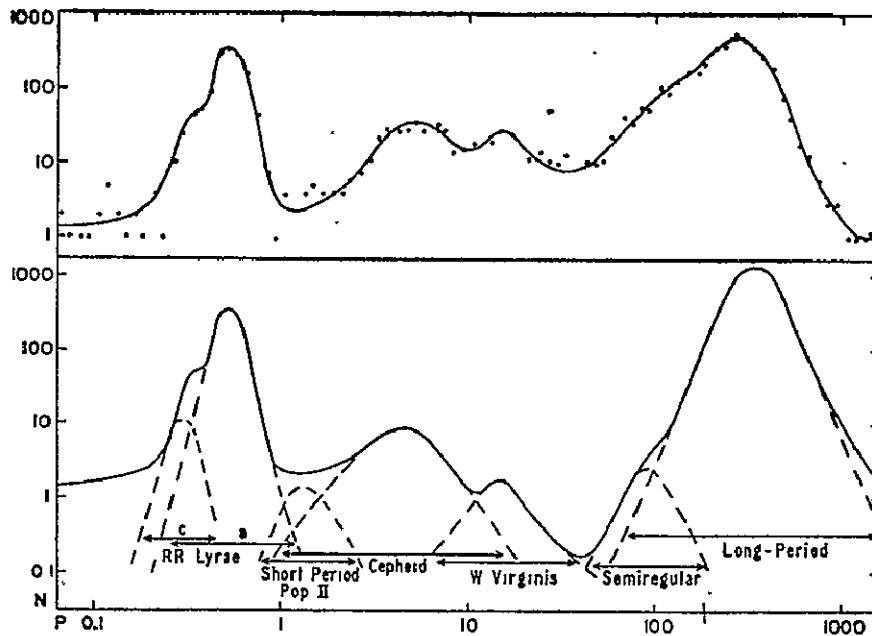


FIGURE 1. FREQUENCY OF VARIABLE OCCURRENCE

"Observations of V and B-V are presented for 45 cepheids brighter than 12.7 photographic magnitude at maximum in a 50 degree section of the northern Milky Way (Lac, Cas, Per, Cam).  $\Gamma$  measurements or estimates of spectral type from classification spectrograms were obtained for all but three stars. Two (one definitely, one probably) were found to be members of population II; distances are given for the remaining 43" [6]. Tables summarizing photometric data, spectral types, color excesses, amplitude defects, and distances for 45 cepheids may be found in References 7 and 8. The spectral types of 22 cepheids are given in Table II [9].

## CEPHEIDS — TYPES I AND II

In 1944, Walter Baade [10] concluded that the stars of the universe are divided into two major groupings, or population types. Population I objects were made up of "young, hot, short-lived stars." These were the stars of the spiral arms of the galaxies. Classical (or type I) cepheids were assigned to this group. The fainter and shorter period cepheids (speaking of the RR Lyrae type) that were associated with the globular clusters forming the halo of the galaxies were placed by Baade among the "older, longer-lived stars" of Population II. The W Virginis (or type II cepheid) stars are also Population II.

TABLE II. SPECTRAL TYPES OF TWENTY-TWO CEPHEIDS

Star	Period (days)	Log <i>P</i>	Spectral Type (Code or Struve)	Spectral Type (Present)	Δ <i>Sp</i> (Present)	Δ( <i>B</i> - <i>V</i> ) (Eggen <i>et al</i> 1957)
SU Cas ..	1.95	0.29	F5 I-II—F7 I-II	F5 Ib-II—F6 5 Ib-II	0.15	0.14
DT Cyg. .	2.50	0.40	F5 5 I-II—F7 I-II	F5.5 Ib-II—F7.5 Ib-II	0.2	14
SZ Tau....	3.15	0.50	F6 Ib—F9 Ib	F5 Ib—F9.5 Ib	0.45	17
RT Aur . .	3.73	0.57	.....	F5.5 Ib—G0 Ib	0.45	.37
α UMi. . .	3.97	0.60	.....	F7 Ib-II—F8.1b-II	0.1	.05
T Vul. . .	4.44	0.65	.....	F5 Ib—F9.5 Ib	0.45	30
V 386 Cyg..	5.24	0.72	F5 Ib—G1 Ib	F5 5 Ib—G0 Ib	0.45	...
δ Cep . . .	5.37	0.73	F5 Ib—G2 Ib	F5 Ib—G1 Ib	0.6	.37
MW Cyg . .	5.96	0.78	F8 Ib—G1 Ib	F8.5 Ib—G2 Ib	0.35	...
U Sgr . . .	6.74	0.83	.....	F5 Ib—G1 5 Ib	0.65	39
η Aql .....	7.18	0.86	F6 Ib—G4 Ib	F7 Ib—G2 Ib	0.5	41
VY Cyg . .	7.86	0.90	F6 Ib—G1 Ib	F5 5 Ib—G0 5 Ib	0.5	...
S Sge . . .	8.38	0.92	F6 Ib—G5 Ib	F7 Ib—G3 Ib	0.6	40
BZ Cyg. . .	10.14	1.01	F8 Ib—G5 Ib	F7 Ib—G4 Ib.*	0.7	...
ζ Gem....	10.15	1.01	F7 Ib—G3 Ib	F7.5 Ib—G1.5 Ib	0.4	30
Z Lac. ....	10.89	1.04	F6 Ib—G6 Ib	†	...	51
TX Cyg....	14.71	1.17	F5 Ib—G6 Ib	F5.5 Ib—G5 Ib:	0.95:	...
SZ Cyg....	15.11	1.18	F8 Ib—G8 Ib	F7.5—F8 Ib—G6 5 Ib	0.9	...
X Cyg . . .	16.39	1.21	F7 Ib—G8 Ib	F7 Ib—G9 Ib:	1.2:	64
CD Cyg . .	17.07	1.23	F8 Ib—K0 Ib	F8 Ib-Iab—G5 Ib-Iab	0.8	...
T Mon . . .	27.01	1.43	F7 Ia-Ib—K1 Ia-Ib	F7 5 Ib-Iab—G8 Ib-Iab	1.05	55
SV Vul. . .	45.13	1.65	F7 Ia—K0:	F7 Iab—K0 Iab:	1.3:	0.70

\* Colons indicate uncertainty.

† In the writer's opinion, there are not enough spectrograms to make a satisfactory determination of spectral types.

The classical cepheids are so designated because they resemble their prototype,  $\delta$  Cephei. Approximately 600 are known in our galaxy, where they congregate near the plane of the galaxy, and throughout the spiral arm structure. The classical cepheids are yellow supergiants, and the brightest classical cepheids visible to the naked eye are Polaris, Delta Cephei, Eta Aquilae, Zeta Geminorum and Beta Doradus. Polaris has the smallest amplitude (light variation) of all, which is 0.17 magnitude in the ultraviolet and 0.04 magnitude in the infrared.

The type II cepheids, or W Virginis stars, so named because they resemble their prototype, W Virginis (the brightest member of the group), have been recognized more frequently in the globular clusters and near the center of our galaxy. The periods of variation are generally between 12 and 20 days. The light curves have broader maxima and are more nearly symmetrical than classical cepheids. Also, they fall from maximum to minimum light more slowly, and are somewhat bluer in color.

The cepheids are highly luminous stars. Polaris, the nearest of them, is not a particularly bright cepheid, but it is about 600 times brighter than the sun (absolute brightness). The brightest Population I cepheids are more than 10 000 times as luminous as the sun.

Differences in the forms of the light curve are not the only distinctions between type I and type II cepheids [2, p. 1143]. The spectra of the type II cepheids differ from those of the classical cepheids. Seventy percent show bright lines of hydrogen. Also, the hydrogen lines reach their greatest intensity as the brightness of the star increases. When compared to classical cepheids of the same periods, the spectral types of the W Virginis stars are earlier, especially at minimum light, and the hydrocarbon (G) band is not found in this spectrum. They show bright hydrogen lines, particularly on the rising or ascending wing of the light curve, a property which is not displayed in the classical cepheid.

The spectrum of W Virginis has been studied in detail by Sanford and Abt [2, p. 1452]. "The absorption lines are double for a short time around maximum, and this has been interpreted as being due to successive waves proceeding outwards from the deeper layers of the star's atmosphere; a new wave would start shortly before the previous one has been dissipated. Sanford discussed these phenomena in terms of shock waves passing through the star's atmosphere. The shock wave is known for being a very efficient mechanism for heating and compressing the gas through which it passes, and this would allow sufficient energy to be available in localized regions to produce the emission lines."

Another striking feature of the behavior of the intrinsic type II variables is that their velocity curves are discontinuous.

The most fundamental difference between the cepheid types I and II is age [11]. This implies a difference in mass and chemical composition. W Virginis stars, fairly old and luminous, must have relatively low mass ( $1.2 \odot$ ), while the classical cepheids have masses of at least an order of magnitude larger. Judging by the globular clusters in which they appear, the W Virginis stars may have metal abundances deficient with respect to the sun by factors from 20 to 200. The classical cepheids show a range in chemical composition.

Assuming that mass differences are more important than composition differences, we can see whether the difference in luminosity between the two types of cepheids can be explained by their difference in mass [12].

Assume that the two types of cepheids have the same  $Q$  in the  $P\rho^{1/2} = Q$  (period-mass density) relation. Therefore, this relation can be written

$$P^2 MR^{-3} = Q^2 \quad (M \text{ is the mass})$$

If we then compare a classical cepheid and a W Virginis star having the same period, then  $R \propto M^{1/3}$ . From their relative positions on the H-R Diagram, the two stars presumably do not differ greatly in effective temperature. Thus,  $R \propto L^{1/2}$  and  $L \propto M^{2/3}$ . Suppose, as a typical example, a comparison of the two types of cepheids at a period of 15 days is made. Then the mass of the classical cepheid is about  $10 \odot$ , and that of the W Virginis star about  $1.2 \odot$ , and their luminosity ratio  $(10/1.2)^{2/3}$ . This implies that the classical cepheid should be brighter than the W Virginis star by 1.53 magnitudes, as is approximately observed. It therefore seems likely that the lower luminosities of the W Virginis stars with respect to the classical cepheids can be explained in terms of their differences in mass without invoking the effects of differing chemical compositions.

A re-evaluation of the absolute magnitude of nearby cepheids as a function of period was undertaken by Baade at Palomar, by Thackeray and Wesselink at Pretoria, and by Mineur, Blaauw and H. R. Morgan [13]. They arrived independently at the conclusion that the classical cepheids are about 1.5 magnitudes brighter than had been supposed.

"Violet-displaced emission lines have been observed in the wings of the strong H and K absorption lines of Ca II in several classical cepheids during part of the time of increasing light. In some stars the emission lines are double, the violet component being stronger than the red. Herbig has suggested that the layer producing the emission lines may lie below an absorbing layer. He has also observed weak central emission components in the H and K lines in  $\eta$  Aquilae as well as the stronger displaced emission components. The components which are contributing are as follows: (a) a normal broad photospheric absorption line, (b) a fairly broad emission component supposedly produced in a localized region within the atmosphere, (c) an absorption component arising from the emission line region and (d) a weak emission component which may arise in a region above all other component H-emission lines is not observed in classical cepheids, yet atmospheric motions of considerable amplitude occur. Perhaps this is connected with the coherence of the motion and

absence of large-scale motions of one layer relative to another, such as may occur in the other variables. However, only a detailed study based on knowledge of the structure of such an atmosphere in a steady state will enable these ideas to be tested" [2, p. 1453].

Recent studies [14] of the emission lines of Ca II in classical cepheid variables have resulted in the following:

1. The temporary presence of shortward displaced emission lines of Ca II appears to be a general characteristic of classical cepheid variables, at least those of Eggen's types A and B, with periods of 5 to 16 days.

2. Calcium emission was found for a portion of the time on the rising branches of their light curves.

3. The emission line in H is often equal in intensity to or stronger than that in K.

4. Nothing was found in the material examined that did not fit in with the interpretation of the emission-line phenomenon, in terms of the self-absorption of a broad, displaced emission line by an overlying cooler gas.

"Certain absorption lines in the spectrum of W Virginis have been found to be double during a short phase interval centered approximately at light-maximum. They behave as if a sequence of velocity variation starts abruptly with the appearance of lines showing high velocity of approach, just before light maximum and while the lines of the preceding sequence showing high velocity of recession are still present. As the sequence continues, the lines, while at first showed high velocity of approach, move longward until they in turn show high velocity of recession. They do not disappear until after the next light-maximum, by which time another velocity sequence is well advanced. The duration of a velocity sequence is thus from shortly before one light-maximum until shortly after the next" [15].

"It is perhaps significant that two Sr II lines  $\lambda\lambda$  4077 and 4215 and the strong Ti II lines which are most conspicuously double in the spectrum of W Virginis are also the lines that widen and have velocity displacements in certain other cepheids at or near light-maximum" [15].



## THE LIGHT CURVE

Three quantities uniquely determine a star in space: position, radiation, and time. Tremendous usage has been made of the latter two quantities in the study of variable stars. A variable star then is studied by analyzing its spectrum and by measuring the variation of its light with lapse of time.

The three techniques commonly employed to measure the star's light intensity are photographic photometry (emulsions), photoelectric photometry (photomultipliers), and photoconductive photometry (PbS). A fourth method used in early photometry is photovisual photometry.

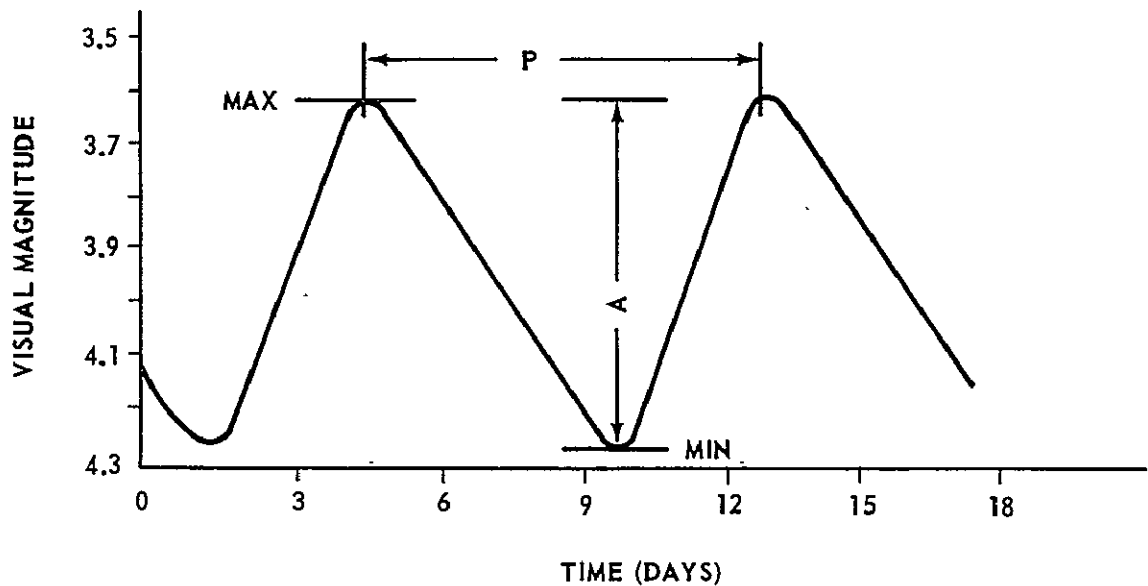
The principal source of information that shows how a variable star's brightness changes with time is the light curve, a graph (or correlation) of apparent magnitude plotted against the time. To obtain a light curve, you would use one of the techniques listed above, i. e. , photograph the star at certain frequent intervals, or compare its brightness (photoelectrically, visually, or conductively) with neighboring (standard) stars. The light curve is then obtained by drawing a smooth curve through all these single magnitudes (observations). An example of a light curve is given in Figure 2.

Figure 2 shows that the elements of the light curve are the period, the amplitude, maximum brightness, and minimum brightness. Another element of the light curve is the epoch, or time of a well-defined place on the curve. As an example, the elements for the cepheid variable star Eta Aquilae ( $\eta$  Aql) [16] are:

$$\text{Maximum brightness} = 2\,414\,827.15 + 7.1767\,E \text{ (occurrence),}$$

where the first number is the epoch expressed in Julian days and the second is the period in days.  $E$  expresses multiples of the period ( $E = 1, 2, 3, \dots$ ). The Julian Day is the number of days that have elapsed since the beginning of the arbitrary zero day at noon Greenwich Mean Time on January 1, 4713 B.C. It is a device often used in astronomical records to avoid the complexity of the calendar system.

The light curve of a pulsating star is the consequence of two sources [17]. The main contribution to the variation comes from the changes in emission/cm<sup>2</sup> of the stellar surface, which can be called the emissivity effect. This effect is caused by changes in the temperature and the density of the



**LEGEND:**

- P:** PERIOD, TIME BETWEEN SUCCESSIVE MAXIMA.
- A:** AMPLITUDE, DIFFERENCE BETWEEN MAXIMUM AND MINIMUM LIGHT (THE RANGE OF MAGNITUDES).
- MAX:** POINT OF MAXIMUM LIGHT
- MIN:** POINT OF MINIMUM LIGHT

**FIGURE 2. LIGHT CURVE OF A TYPICAL CEPHEID VARIABLE (TYPE A)**

atmosphere of the variable star. The other source of the light curve is the result of changes in surface area of the star. This effect can be indicated as the area effect.

For classical cepheids; the light variations are mainly results of changes in effective temperature (emissivity effect). The relative radius amplitude increases slowly with period. For the type II cepheids, the relative radius amplitude increases rapidly with period until, for stars of the longest period W Virginis (henceforth designated WV stars) type, the variation in radius is of the order of the mean radius of the star itself [18].

There is a general tendency for the light curves of stars of similar period to be alike, and there is a progression in the form of light curve with

period. Hertzsprung (1926) pointed out that galactic cepheids display this progression in the form of light curve with period. The WV stars are excluded. The progression is in the form of the light curve, as indicated by asymmetry and the occurrence of humps, but there is considerable dispersion in amplitude at any one period. This progression in form of light curve with period is illustrated in Figure 3 [5, p. 33].

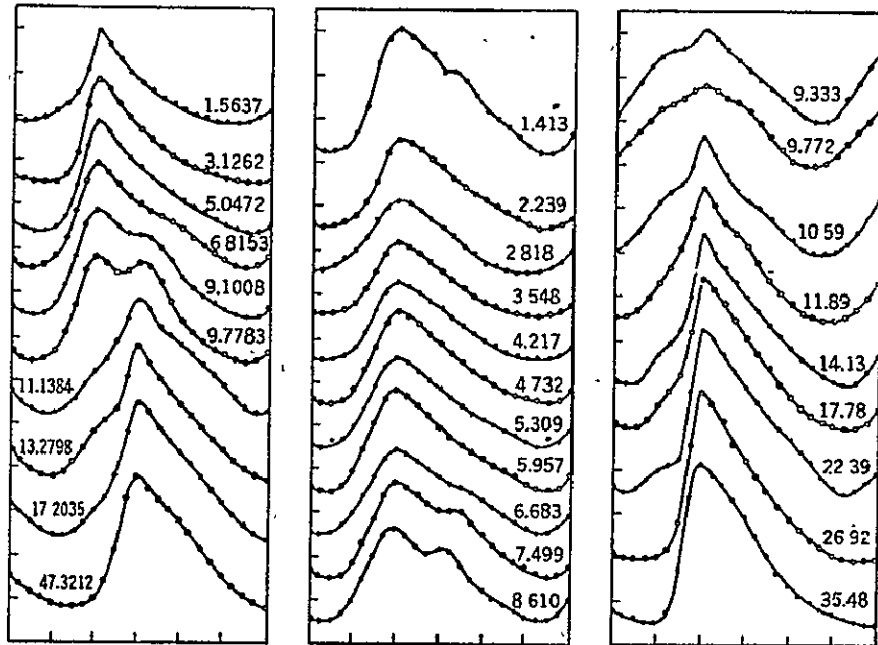


FIGURE 3. RELATION BETWEEN PERIOD AND LIGHT CURVE FOR CLASSICAL CEPHEIDS (THE HERTZSPRUNG RELATION)

"The different types of light curves grade into one another, in general, in a steady progression (the Hertzsprung Relation). The progression runs through smooth asymmetrical light curves; curves with a hump on the downward branch; curves with two nearly equal humps; roughly symmetrical curves with a more or less pronounced central maximum; highly asymmetrical curves with a subsidiary rise just before the main rise; and finally, smooth asymmetrical light curves not unlike those shown at the short periods. These curves occur in a more or less steady progression with increasing period" [19]. There is a continuous graduation of types from the shortest to the longest periods. "There is a general tendency for a given type of light curve to occur within a given range of periods; there is also a less marked relationship with apparent magnitude; fainter stars reach a given curve type at shorter periods than brighter stars" [19, p. 185].

Recent work [19, p. 186] suggests that a cepheid variable is moving from left to right in the H-R plane having left the Main Sequence and not yet reached the Red Giant stage. If, then, an examination of a series of light curves for stars of the same apparent magnitude and increasing period is made, a trace of the changes of the light curve and the uses of them as clues to the physical changes of the star could possibly be made [20].

Another important result [21] from the cepheid light curve was the relation discovered by Parenago and Kukaskin (1936), who found that most of the characteristics of the classical cepheids undergo discontinuous changes at a period of about 10 days. These relationships between shape of light curve and length of period have been well summarized by Payne Gaposchkin (1954) [21].

"In a previous paper (Eggen 1951) the cepheids were divided into three groups, A, B, and C, on the basis of the relationship between light and color, amplitudes and periods. From the material then available it was found that this classification, in general, divided stars into (i) those with  $\delta$  Cephei-like or smooth, asymmetric light curves — type A; (ii) those with  $\eta$  Aql-like or asymmetric light curves showing a secondary hump — type B; and (iii) those with a nearly sinusoidal (symmetrical) light curve, similar to  $\zeta$  Geminorum — type C. If the maximum follows the minimum by an interval of more than  $0.37^p$ , the star will be assigned to type C; if less, to types A or B according to whether humps are absent or present, respectively" [21].

When color amplitudes and light amplitudes are plotted against the logarithm of the periods, the following is found: "(i) the cepheids of type C have generally smaller amplitudes than those of types A and B; (ii) type A variables greatly predominate in the period range 3.5 to 6.5 days, and type C in range from 9 to 12 days; (iii) all the type A stars in the period range from 3.5 to 6.5 days have greater light amplitudes than those of types B and C" [21, p. 424]. The light curves of typical A-, B-, and C-type cepheids are given in Figure 4a, 4b, and 4c, respectively [21, p. 425].

## THE VELOCITY CURVE

In 1894 Belopolsky discovered that  $\delta$  Cephei has a radial velocity that varies in the same period as the light and in such a way that maximum radial velocity nearly coincides with minimum light and vice versa. A graph that

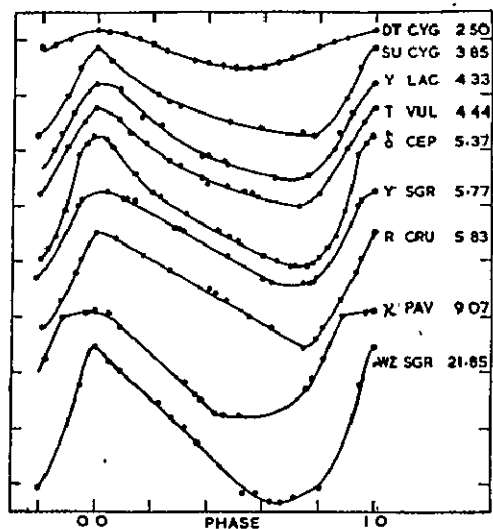


FIGURE 4a. TYPE A LIGHT CURVES

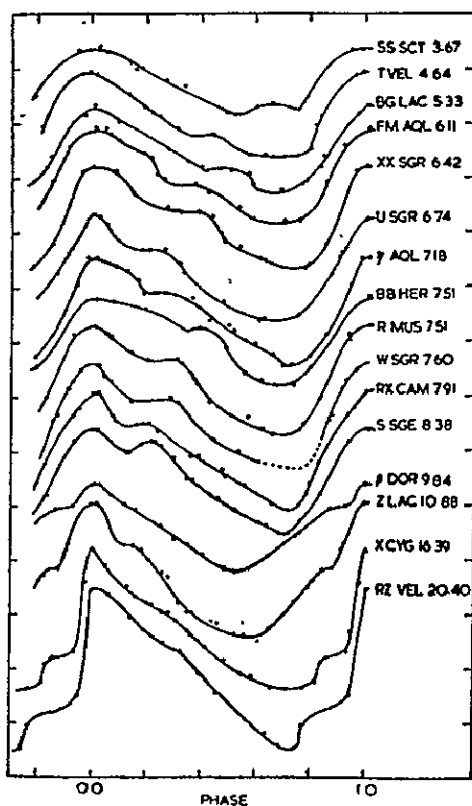


FIGURE 4b. TYPE B LIGHT CURVES

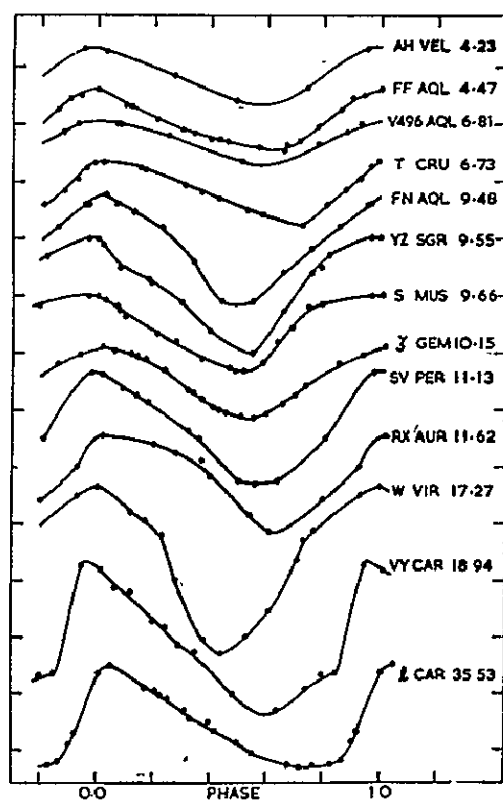


FIGURE 4c. TYPE C LIGHT CURVES

displays changes in the Doppler shifts of the spectral lines of a cepheid, with lapse of time, is called a radial velocity curve, or, simply, velocity curve. A comparison of the two curves, light curve and velocity curve, is given in Figure 5 [1, p. 435].

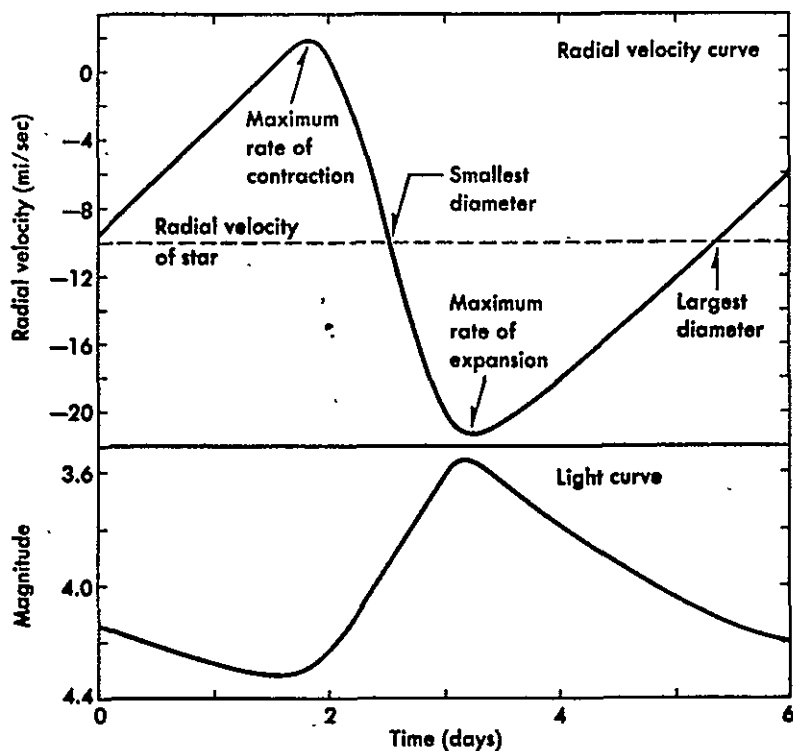


FIGURE 5. COMPARISON OF VELOCITY CURVE AND LIGHT CURVE

It is seen from Figure 5 that the light and velocity curves are quite similar, but this correspondence between the two curves is only approximate. The differences between the two depend on the part of the spectrum that is examined; also, with increasing period of the variation the greatest velocity of approach tends to lag behind the light maximum, as A. H. Joy has shown [17, p. 388].

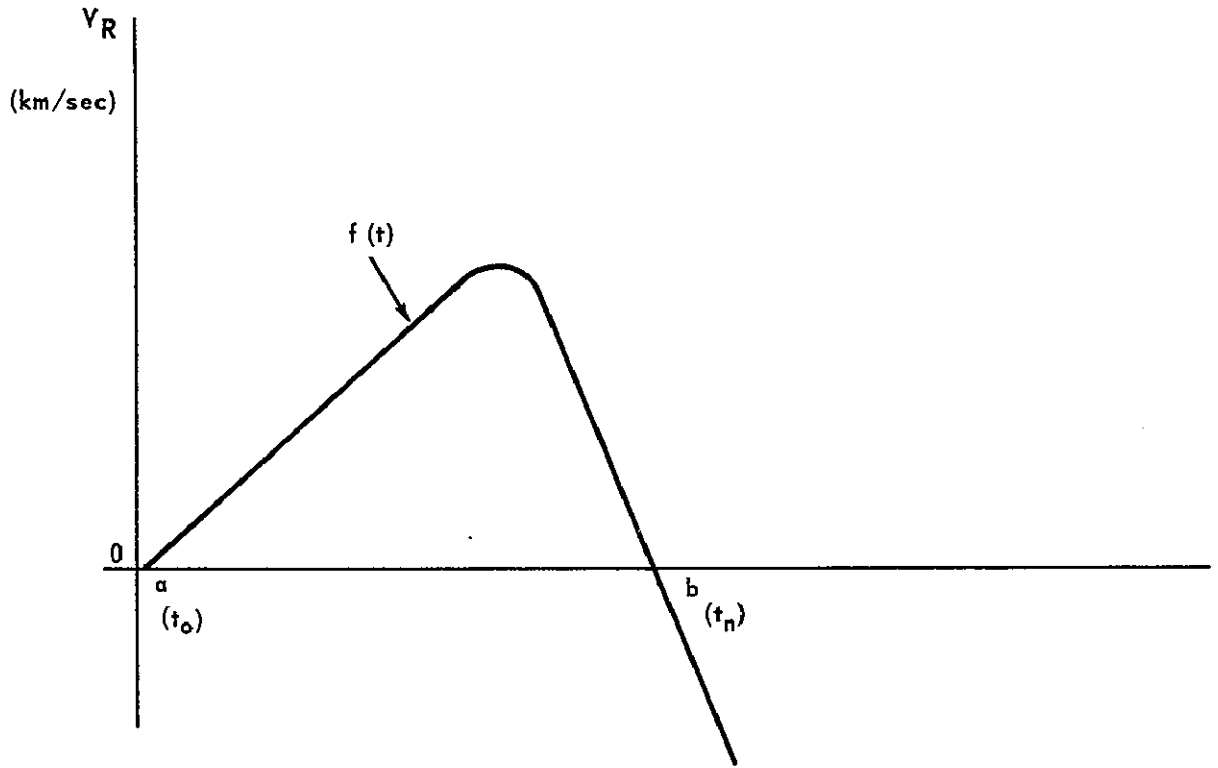
In 1899, K. Schwarzschild found that changes in luminosity of cepheids are accompanied by changes in effective temperature. The temperature at maximum brightness is higher than at minimum brightness (about 1000°K hotter at maximum). Schwarzschild illustrates this by the use of photographic

measurements (the use of extra-focal images). "By placing the plate at some distance, inside or outside the focus, he obtained, for all the stars, smooth discs of equal size but of different blackness, i. e., density of the silver deposit. Differences in blackness can be distinguished far more accurately than differences in diameter on focal plates" [22]. The photographic measurements showed a range in brightness  $1\frac{1}{2}$  times larger than was observed visually, indicating a color more blue at maximum than at minimum.

Earlier (1879), the German physicist August Ritter had suggested that radial pulsations, accompanied by variations in surface temperature, might be responsible for the variability of some stars showing periodic variations of light. However, it appears that Shapley was the first astronomer to propose the pulsation hypothesis as a definite substitute for the binary star hypothesis which was clung to for many years, basing the suggestion on an analysis of observed facts (1914).

The Doppler shifts result from the periodic rising and falling of radiating surface rather than the orbital motion of the star as a whole. The mean value of the apparent radial velocity corresponds to the line-of-sight motion of the star itself; the photospheric pulsations cause variations about this mean value. When the photosphere expands, it is approaching us with respect to the rest of the star, and each spectral line is shifted to slightly shorter wavelengths than that of its mean position; when the photosphere contracts, the lines are shifted to slightly longer wavelengths. When the photosphere reaches its highest or lowest point, that is, when the star is at its largest or smallest size, these spectral lines are not displaced from the positions that correspond to the radial velocity of the star itself.

An approximation method that can be used to calculate the total distance through which the cepheid's photosphere rises or falls, i. e., the change in radius,  $\Delta R$ , can be found by using the velocity curve. It is noted that by multiplying the velocity of its rise at each point in the pulsation cycle by the time it spends at the velocity, and then adding up the products (technically, by integrating the velocity curve over the time of rise or fall) will give you a numerical approximation to  $\Delta R$ . This is more or less an extension of the trapezoidal approximation rule. This method is outline as follows:



$$\begin{aligned} \Delta R &= \int_a^b f(t) dt = \frac{b-a}{2n} \left[ f(t_0) + 2f(t_1) + 2f(t_2) + \dots + 2f(t_{n-1}) + f(t_n) \right] \\ &= \frac{b-a}{n} \left[ f(t_1) + f(t_2) + \dots + f(t_{n-1}) \right] \end{aligned}$$

since

$$f(t_0) \text{ and } f(t_n) = 0.$$

By knowing the star's actual radius  $R$  and its  $\Delta R$ , it is possible to calculate percentage of change in  $R$  for the star over the pulsation cycle. For most cepheid variable stars the percentage of change in  $R$  is about 7 or 8 percent. For example, in the case of  $\delta$  Cephei, it is found that the photosphere pulsates up and down over a distance of about  $2.7 \times 10^6$  km and that the mean radius  $\bar{R}$  of  $\delta$  Cephei, computed from its absolute magnitude and temperature, is about  $4 \times 10^7$  km; hence, the change in the radius of  $\delta$  Cephei is of the order of 7 or 8 percent of the radius [3, p. 339].



The cepheid variables known in our galaxy are intrinsically very luminous stars. Russell and Hertzsprung [23] independently showed that the Cepheids are stars of small peculiar motions and small parallaxes, and, hence, of great absolute brightness. Russell found a mean absolute magnitude of  $-2.4$  and Hertzsprung found one of  $-2.3$ , i. e., the average cepheid (the spectrum is of solar type) is nearly 700 times as bright as the sun. It is reasonable to assume that the cepheids and the sun have a comparable surface brightness. This would imply that the average cepheid has a volume between 15 000 and 20 000 times that of the sun. Because of its high luminosity, and consequently of its large mass, the cepheid variable is probably uncommon in space.

"In 1926, Baade remarked that pulsation theory of cepheid variation could be tested with measures of brightness, color, and radial velocity. The test, if successful, would lead to a determination of the mean radius  $\bar{R}$  of the variable star in absolute measure. The argument is essentially as follows:

"If black-body radiation is assumed for the radiation of the variable, the surface brightness may be computed from the observed color; the area then follows in terms of an arbitrary unit, by dividing the observed light by the surface brightness. Hence, the radius becomes known as a function of the phase, in an arbitrary unit. On the other hand, the displacement of the star's surface may be found in kilometers by integrating the radial velocity curve. It is clear that the radius and the displacement so determined should be in phase and that if they prove to be so, the mean radius can be calculated since a known fraction is known in absolute measure" [24].

Bottlinger [21], in 1928, made the first attempt to determine  $\bar{R}$  by the above method with observations of  $\zeta$  Geminorum. His attempt was unsuccessful. Bottlinger then concluded that the assumption of black-body radiation was fallacious. It is well known from investigations on constant stars, that different parts of the spectrum lead to different color temperatures, a fact which shows that the stars do not radiate as black bodies. It was found that Baade's suggestion cannot be carried out unless one is able to put the calculation of the surface brightness on a sound basis which evades the assumption of black-body radiation.

Later Wilhelm Becker [24] replaced the assumption of black-body radiation by the hypothesis that among cepheids and throughout their variation, there exists a single-valued relation between color and surface brightness. From this beginning, A. J. Wesselink formulated a method for determining stellar radii. His method is as follows:

Assume that "the C.I. (color index) of a star is a unique, monotonic function of its effective temperature, and that the B.C. (Bolometric Correction) is dependent on the C.I. by another such function. Thus when, at two different phases, the star has the same C.I., it also has the same effective temperature. But, in general, the apparent magnitudes of the star at those two phases will not be equal, and the difference between them (which is equal to the difference in bolometric absolute magnitude) will yield the ratio of the luminosities at those two phases. Since at constant temperature the luminosity ratio is equal to the square of the ratio of the radii at the two phases, this ratio,  $R_1/R_2$ , may be obtained. Furthermore, the velocity curve of the variable can be integrated between the two phases to give the difference in radius ( $R_2 - R_1$ ). Hence,  $R_1$  and  $R_2$  can be obtained individually. Integration of the velocity curve also gives the differences between  $R_1$  and  $R_2$  and the mean radius  $\bar{R}$ , and so yields  $\bar{R}$ " [24].

"The validity of Wesselink's method rests on the basic assumptions (a) that at two points on the light curve where the colors are equal the effective temperatures are also equal; and (b) that actual radial displacement may be obtained from the observations of radial velocity" [18, p. 85].

J. Stebbins has done a great deal of work in 6-color photometry and radii determinations. Following Stebbins, one can write the luminosity of the cepheid at any phase [18, p. 85] as

$$L_R = 4 \pi \sigma (R_M + \Delta R)^2 T_e^4$$

where

$\sigma$  = Boltzmann's constant

$R_M$  = Minimum Radius

$T_e$  = Effective Temperature

The ratio of light at any two phases of equal color (equal color implies same  $T_e$ ) is given by

$$L_1/L_2 = (R_M + \Delta R_1)^2 / (R_M + \Delta R_2)^2$$

and if  $M_1$ ,  $M_2$  are the observed magnitudes corresponding to  $L_1$ ,  $L_2$  then the minimum radius of the star may be written

$$R_M = (\Delta R_1 - A \Delta R_2) / A - 1$$

where  $A = \text{antilog} \left( \frac{M_2 - M_1}{5} \right)$ .

It can be shown that the minimum radius of the cepheid at any two phases is given by

$$R_M = \frac{\Delta R_1 \left( \frac{T_{R_M} + \Delta T_1}{T_{R_M} + \Delta T_2} \right)^2 - A \Delta R_2}{A - \left( \frac{T_{R_M} + \Delta T_1}{T_{R_M} + \Delta T_2} \right)^2}$$

where  $A$  is equal to the same  $A$  above.

Wesselink's method is a powerful one, and not the least of its attractions is that neither the distance nor the reddening of the star need be known. Nevertheless, over the years a number of difficulties have been encountered in its application. It has been pointed out that some of the negative results obtained by Wesselink's method may result from insufficient attention to elementary considerations [25], in particular to correct phase matching of the variations in light, color index, and velocity.

Ideally, one should have simultaneous spectroscopic and photometric observations [25]. But in practice, however, "these two sets of observations are usually made by different observers at different places at different times. In order to apply Wesselink's method what is usually done is simply to use the best available period of the star to bring one set of observations up to epoch of the other set" [25]. One should not rely on a given period to an accuracy greater than about one part in  $10^4$ , at least when dealing with classical cepheids [25]. This limit is set by observational errors and by physical instabilities in

the star itself. "If the general run of cepheid periods are not to be relied upon to better than about  $1 : 10^4$ , then a mismatch in phase of  $0^{\text{P}}.05$  is likely to occur if the interval separating the spectroscopic and the photometric observations is about 500 cycles" [25, p. 97].

"As a rule of thumb, it appears that if the radius is required with 10 percent accuracy, the relative phases of the photometric and radial velocity curves must be accurate to about 1 percent, which in turn implies that no more than about 100 cycles should have elapsed between the epochs of the photometric and radial velocity measurements" [25, pp. 98-99].

"Although in principle only one pair of equal-color phases is required to determine the radius of the variable, in practice several such phase-pairs may be used, each yielding a value of the mean radius. Generally these values differ somewhat due to random errors in drawing the light, color index, and radial velocity curves, and the adopted value is simply their average. If, however, there is a mismatch in the phases of the curves, then there will be, in addition to the random variations, strong systematic variations" [25, p. 99].

The procedure is to match the curves in phase as well as possible by use of the published period. The analysis is then carried out for six pairs of equal-color phases. The average of the six values for the mean radius is taken, and the sum of the squares of the deviations of the individual values  $\Sigma v^2$  is computed. In effect, one is computing the probable error of the average. The velocity curve is then arbitrarily shifted in phase by some small amount, say  $0^{\text{P}}.02$ , and the entire analysis repeated. After this has been done a sufficient number of times, a plot of  $\Sigma v^2$  against average radius is made and a curve is obtained. The minimum on the curve is the best value that can be deduced from the data [25, p. 99]. The correct match in phase is not far from where the maximum of the light curve is close to the minimum of the velocity curve, or the second minimum may correspond to a radius quite different from the expected value. It is found that the use of approximate differential expressions in Wesselink's method [25, p. 101] (such as  $\bar{R} = 2.17 \Delta R / \Delta M$ , where  $\Delta R$  is the change in radius between two phases of equal color, and  $\Delta M$  is the difference in apparent magnitude between the same two phases) leads to results that are systematically in error by up to 10 percent.

Other important relationships for cepheids include the period-spectrum relation, the period-luminosity relation, the period-luminosity-color relation, the  $P\sqrt{p}$  relation of Eddington (giving use to well-founded ideas concerning the pulsation theory of variable stars), and the H-R Diagram. These will be the topics of the next section.

## OTHER RELATIONSHIPS

If the periods of the intrinsic variables are plotted against their spectral types, a relation known as the period-spectrum relation is observed. This tells the viewer that the spectral type (or temperature) of the stars varies in time as the light curve. As an example, the colors of  $\eta$  Aquilae ( $P = 7.^d.1766$ ) at maximum nearly match those of supergiant stars of spectrum cF7; at minimum, cG3, not quite a typical star at either phase. Or for Polaris, ( $P = 3.^d.96$ ) the inferred variation of spectral type is from cF6 to cF7. The small "c" in these spectral classes refers to the supergiant stage of which the cepheid variable is a part (the old system of classification).

The most important of all these relationships is the period-luminosity relation (P-L), which can be formulated as follows: If you know the period of a star, you also know its luminosity and, by comparison with the apparent magnitude, its distance. This remarkable and extremely useful P-L relation resulted from the discovery in 1912 by Miss H. S. Leavitt of Harvard that a group of variable stars in the Small Magellanic Clouds (SMC) show a correlation between their periods and apparent brightness. She found the periods of the stars were related to their relative brightness, in the sense that the brighter-appearing ones always had the longer periods of light variation. The P-L relation exists as a linear relation between the median absolute magnitude and the logarithm of the period (the median magnitude is the average between the magnitudes at maximum and minimum brightness). First established in useful form by Shapley in 1917, the P-L Relation has especial importance in providing a cosmic distance scale because these very luminous stars are visible at great distances [26].

A period-luminosity relation seems to hold for both types of cepheids — classical-type I cepheids and WV-type II cepheids. In both cases it is found that stars of longer periods are the more luminous.

The P-L relation for both types of cepheids must be represented by bands at least one magnitude wide rather than by simple curves. This modification resulted from H. C. Arp's studies of cepheids in the SMC, and was explained by A. R. Sandage. From a relation already known between the period and the mean density, he deduced that the absolute magnitude of a cepheid depends not only on the period of its pulsation but also on the star's mean surface temperature or color. This would apply to all pulsating stars.

The astronomical yardstick is the cepheid variable star, the principal measure for plumbing the depths of space outside the Milky Way. Walter Baade, at the meeting of the International Astronomical Union in Rome in September of 1952, announced the doubling of the yardstick [27]. Baade's conclusion and Sandage's evidence (the color-luminosity relation) implied the following: The error of 1.5 magnitudes, corresponding to an error in the resulting distances by a factor of 2, in the sense that all distances must be increased by this number, is attributable to the classical cepheids. The curve representing their intrinsic P-L relation should apparently be raised 1.5 magnitudes above that for other cepheid-like pulsating variables, such as the WV stars.

The former curve for the classical cepheids was therefore raised 1.5 magnitudes in the diagram and was replaced at the lower level by the curve for the type II cepheids. The horizontal line for the RR Lyrae variables was left unaltered.

The error in the P-L relation was caused by inaccuracies in the selection of the zero point of the P-L relation. Therefore, how was the zero point determined and why was the zero point of the classical cepheids in error by as much as 1.5 magnitudes ?

"The apparent angular motions of bright cepheid variables have been determined from accurate measures of their positions in the sky extending over hundreds of years. For the same stars, radial velocities have been determined from measures of the Doppler shifts of their spectral lines. An individual cepheid, however, may be moving only at right angles to the line of vision, thus having large proper motion and no radial velocity with respect to the earth. Another star could be moving entirely along the line of sight, and would then have no detectable proper motion, or tangential velocity. It is not possible to determine in the case of an individual star whether a small proper motion results from an actual small velocity at right angles to the line of sight, or whether the motion seems small because the star is very far away. But, if the problem is considered statistically, the motions may be assumed to be at random, the radial component being as large as the tangential. The radial and linear tangential values may roughly be equated. On this basis, the measured radial velocity should be approximately a measure of the star's linear motion, while the angular proper motion should be a measure of the same velocity as seen from the distance between the earth and the star. Thus, if a star of large radial velocity has a small proper motion, its distance must be relatively great; stars of larger proper motion must be nearer. It is important to realize that if, over a large statistical sample of stellar data, the similarity of the radial and tangential velocities is granted, relatively

reliable statistical parallaxes may be obtained. In this manner, Shapley set the zero point of his famous period-luminosity curve" [28].

"It is now known that interstellar absorption tends to make stars look fainter than they otherwise would. A connection to the observed apparent magnitude therefore has to be introduced, allowing for intervening dust and gas, and thus reducing the distance somewhat. This factor was not taken into account in Shapley's early work" [28, p. 325].

The framework of the metagalactic system rested entirely upon Shapley's zero point of the period-luminosity curve; numerous attempts were made to revise the original value, and various corrections to Shapley's zero-point determination were indicated. The range covered by these corrections was about 1.5 magnitudes, indicating among other things how difficult it is to derive the zero point accurately.

The next question to be considered is why the zero point of the classical cepheids was in error by as much as 1.5 magnitudes. "In Shapley's early work it had not yet been realized that the cluster type variables belong to Population II (old stars), while most classical cepheids are members of Population I (young stars). Hence, in the earlier discussions there was a tendency to force the P-L relation to form a continuous curve from the shortest periods of two hours to the longest periods of 40 to 50 days" [28, p. 329]. Also, there was the problem of interstellar absorption.

Baade's work was confirmed by the independent results of A. D. Thackeray. It would appear that there is near unanimity among the active workers in this field now that zero point of the P-L relation has been determined fairly accurately and is close to Baade's revised value [28, p. 328].

The earlier papers of the series "Color Excesses for Supergiants and Classical Cepheids" by Robert P. Kraft have dealt with the determination of the period-color and period-luminosity relations for cepheids in the vicinity of the sun. Basically, the method consists of calibrating the spectral type-intrinsic color relation and the period-luminosity law from the intrinsic colors and absorption-free absolute magnitudes of cepheids in galactic clusters. The spectral type-intrinsic color relation is shown in Figure 6 [29]. The method depends on the fact that all cepheids, regardless of period, come into the range F5 Ib to F8 Ib at maximum light, but the spectral types near minimum become later with increasing period (Code 1947).

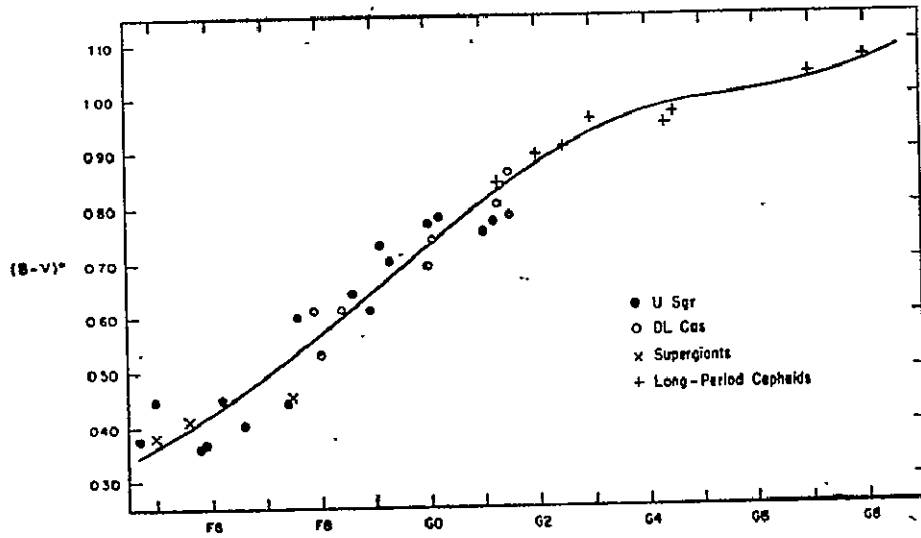


FIGURE 6. SPECTRAL TYPE-INTRINSIC COLOR RELATION

Table III [29, p. 622] is included to show the spectral type-intrinsic color relation for stars of Class Ib.

TABLE III. SPECTRAL TYPE-INTRINSIC COLOR RELATION  
FOR STARS OF CLASS Ib

Sp. Type	(B-V) <sup>0</sup>		Sp. Type	(B-V) <sup>0</sup>	
	Paper II	Present Paper		Paper II	Present Paper
F5 Ib.....	0.36	0.36	G1 Ib.....	0.765	0.80
F6 Ib.....	.42	.42	G2 Ib.....	0.84	0.87
F7 Ib.....	.47	.48	G3 Ib.....	0.91	0.935
F8 Ib.....	.54	.56	G4 Ib.....	0.97	0.975
F9 Ib.....	.62	.65	G5 Ib.....	1.01	1.00
G0 Ib.....	0.685	0.72	G8 Ib.....	1.09	1.06

The spectral type-intrinsic color relation is used to derive excesses for field cepheids. The intrinsic mean color  $\langle (B-V)^0 \rangle_{\text{mag}}$  is plotted as a function of log P in Figure 7 [29, p. 623], in which the five galactic-cluster cepheids are plotted as filled dots. The curve line is an eye-estimate fit and can be represented with a precision of 0.005 magnitude or better by the equation

$$\langle (B-V)^0 \rangle_{\text{mag}} = -0.101 (\log P)^2 + 0.5385 (\log P) + 0.2644.$$



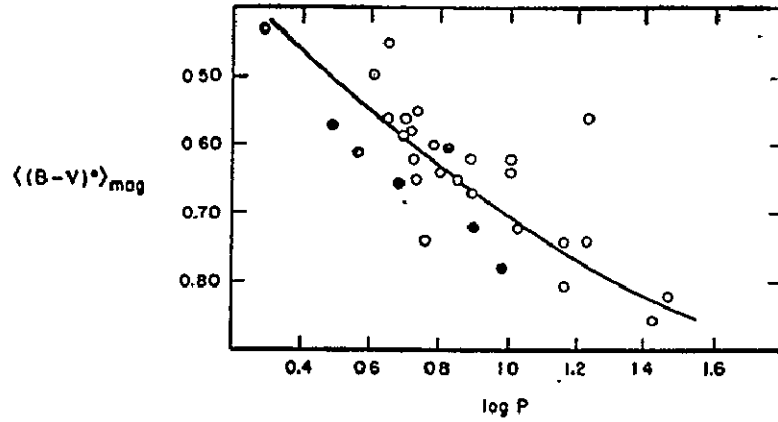


FIGURE 7. THE PERIOD-MEAN COLOR RELATION

The period-amplitude ( $\Delta B$  versus  $\log P$ ) diagram for 54 cepheids confined within 2000 pc (parsec) of the sun [6] is shown in Figure 8 [26, p. 624].

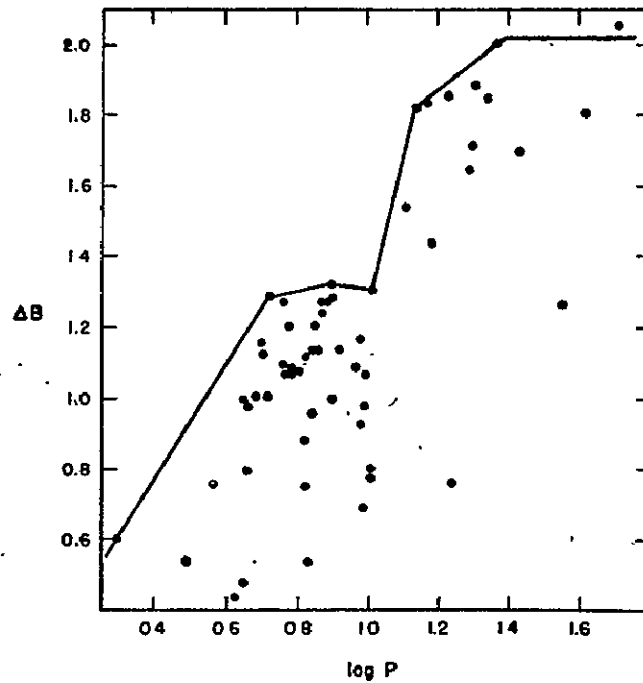


FIGURE 8. PERIOD-AMPLITUDE RELATION

In 1918, Eddington had developed a theory of pulsating stars based upon his researches on the constitution of the stars in general. In this model of a pulsating star the greatest brightness would be reached when the star

attains its smallest size. Its most remarkable achievement was the prediction of a period-density relation for cepheids. Eddington's theory requires that the product of the period and the square root of the mean density be nearly a constant for all pulsating stars, i. e. ,  $P\sqrt{\rho} = Q$ , where  $Q$  is a constant.

"The relation between the period and the density in effect implies that for stars of low average density, the period of pulsation must be lower than for stars of high average density. A giant star like  $\delta$  Cephei has a mean density  $4 \times 10^{-4}$  that of the sun (which is of the order of  $1.4 \times$  the density of water). The period of pulsation of  $\delta$  Cephei is 5.37 days, and the square of the period times the mean density is roughly 1/100, and this fixes the constant of the relation. If one should wish then to predict the period of pulsation for a star of solar density, this constant, 0.01, would be divided by the ratio of the densities of  $\delta$  Cephei and the sun,  $4 \times 10^{-4}$ , and this would give the square of the period of the solar-type pulsating star. The square root then would be 2hr" [28, p. 317].

All pulsating stars whose periods are of the order of days, or even hundreds of days, then must have exceedingly low mean densities.

The accepted theory of stellar structure leads to the expectation that the highest temperature would occur at the time of greatest compression (smallest radius) of the star. This would be the time when the gases are at rest (zero radial velocity) on the descending branch of the velocity curve because just before the stage of greatest compression, the gases are still rushing toward the center of the star, away from the observer. In reality, maximum temperature occurs when the star has the greatest velocity of approach, a fact that was not anticipated in the simple pulsating model. M. Schwarzschild, in 1938, suggested that the star's deep interior pulsates in the manner predicted by this theory, but that in its outermost regions the elements of gas do not all vibrate in unison, causing a lag in the light curve by the observed amount. He suggested that compressional waves run upward through the outer layers, reaching the higher levels later than the lower ones. Greatest compression of the gases in and above the photosphere may occur when the waves are moving fastest. The delay in the time of maximum brightness in light of longer wavelength, as is clearly shown in the case of  $\delta$  Cephei, is consistent with the theory in its newer form [30].

In concluding this chapter, a number of graphs are included. They are Figure 9 [31], the light, velocity, radius, temperature, and spectrum curves of Delta Cephei; Figure 10 [1, p. 439], the P-L relation for type I and II cepheids and RR Lyrae stars; and Figure 11 [32], showing the H-R Diagram with special reference to the cepheid variables.

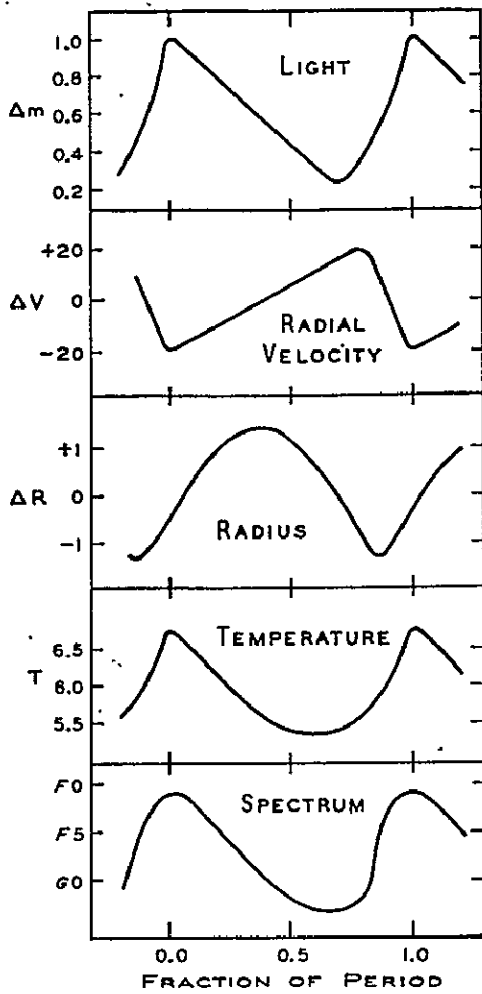


FIGURE 9. COMBINED  
RELATIONSHIPS OF  $\delta$   
CEPHEI

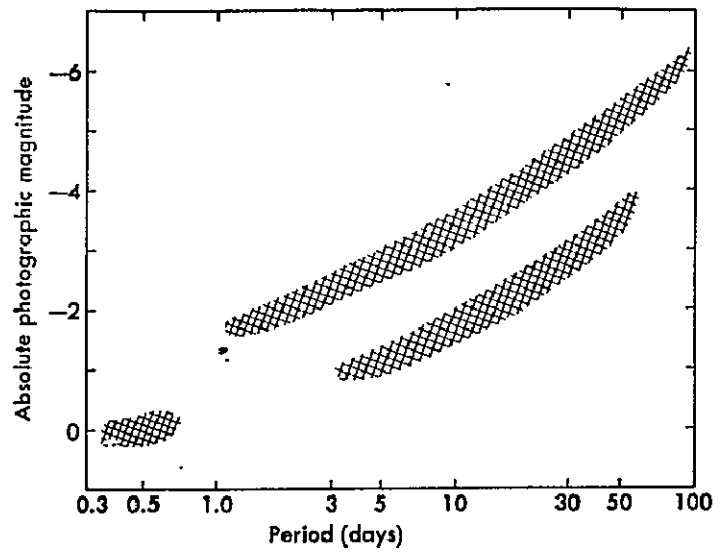


FIGURE 10. PERIOD-LUMINOSITY  
CURVE

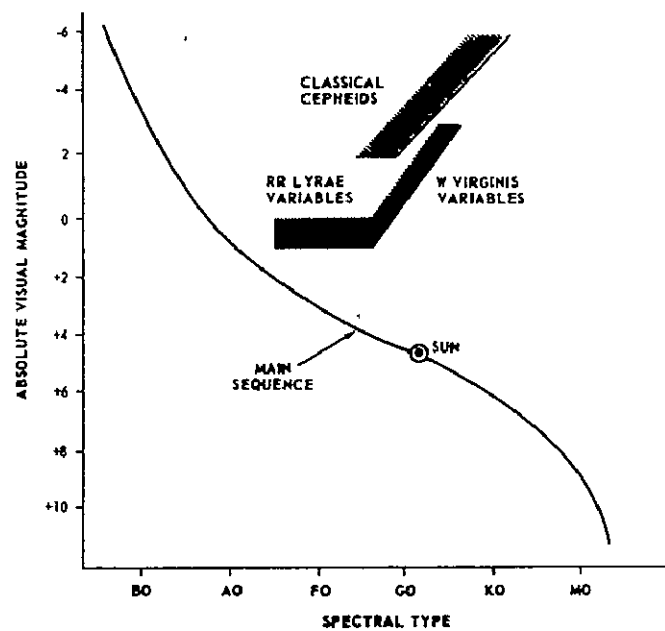


FIGURE 11. P-L CURVE IN H-R  
DIAGRAM

## SPECTRA AND RADIATION CURVES

Each body emits electromagnetic radiation corresponding to its temperature. The distribution of this radiation over the spectrum in each wavelength is given, in the ideal case, by Planck's radiation law [20, p. 8]. With increasing temperature,  $T$ , the total amount of radiation increases, and at the same time, the radiation maximum moves to shorter and shorter wavelengths  $\lambda$  in such a manner that

$$\lambda_{\max} T = \text{Constant (Wien's law)}$$

The constant has the value 0.28979 cm deg [1, p. 355].

"The spectra of cepheids, like those of the great majority of stars, consist of a bright continuous background crossed by numerous dark lines which have been recognized as due to the presence of hydrogen, iron, calcium, etc., in the atmospheres" [33]. Struve [34] states that the spectra of cepheids vary during their light minimum and are practically indistinguishable from those of normal supergiants. Their spectra are rather similar to those of normal supergiants during most of the cycle except at maximum light.

The spectra of cepheids and RR Lyrae stars show two features which are especially significant for the interpretation of these stars [17, p. 387]:

1. The spectral type is variable; from maximum to minimum light, the spectral type advances. Delta Cephei's range is from F4 to G2, signifying a drop of about 1500°K in surface temperature.

2. The spectrum lines oscillate in the period of the light variation. Radial-velocity determinations of the  $\delta$  Cephei type are of especial value for extending our knowledge of the activities taking place in the atmospheres of unstable stars as well as for obtaining data concerning the motions of distant stars and the movement of the galaxy as a whole. The spectra of cepheids are known for their wide deep lines and for the presence of numerous strong lines of ionized atoms.

The hydrogen lines are at their sharpest and deepest at maximum light. Also, the H and K lines are at their sharpest and deepest a quarter period after maximum light; their variation in contour is large [35]. The temperature

is greatest at maximum, and least at minimum. The red and infrared emission and absorption features of stellar spectra assume increasing importance beginning at class G and become the outstanding features of the red stars of classes M, R, N, and S. In particular, the heavy absorption in stars of class M is caused by titanium oxide, whereas that in class S stars is caused by zirconium oxide. The most extensive lists of TiO and ZrO bandheads, with vibrational analyses, are those published by Miss Lowater; Wurm and Meister extended the TiO bands out to 11 238 Å, and Meggers and Kiess described a new system of ZrO lying between 9300 and 9500 Å [36]. It is now almost unanimously recognized that emission lines visible in the light of a star as a whole are caused by the action of diluted radiation in extended atmospheres. It is obvious that on the average the more extended atmospheres have the stronger emission.

"The study of high-dispersion spectra of cepheid variables throughout their light curves is one that deserves to be prosecuted further. It seems likely that a shock wave passes through the atmosphere, and accurate comparison of the behavior of different lines through a cycle can thus throw light on the physical nature of the pulsation near the surface. There is some evidence that velocities differ by as much as three or four km/sec. It has been recently discovered that Ca II H and K lines appear in emission at certain phases in some cepheids" [37].

As stated earlier in this report, August Ritter (1879) suggested that pulsation was the basic cause of the cepheid variability, and since that time no completely satisfactory explanation of the complex phenomenon has been found. Schwarzschild (1935-1938) suggested that standing waves in the interior might become running waves near the surface, and he showed that the flux must, of necessity, be in phase with the velocity and not with the displacement [38]. The cepheid phenomenon may be a transitory disturbance in the life of a supergiant as it evolves across the top of the H-R Diagram. "Two important spectroscopic features of nonvariable supergiants are (1) the H and K emission lines (Wilson and Bappu, 1957) and (2) the increase in the width of the absorption lines with increasing luminosity (Herbig and Spalding, 1955)" [38, p. 371]. Both these features exist in the spectra of cepheids, but are time-dependent phenomena. "The cepheids are recognized tools of galactic research, and it is important that their gross spectroscopic properties — i. e., spectral types and luminosities — be understood in terms of a sequence of spectral types and luminosities among the nonvariable stars" [38, p. 371].

The spectra [38, pp. 371-372] of the classical cepheids are known to be peculiar at maximum light (Struve, 1944) and the peculiarity is the more pronounced, the longer the period. There is some photometric evidence (Gascoigne and Eggen, 1957) that the cepheids have nearly the same colors at maximum, independent of  $P$ . Taken at face value, it implies that the peculiarities of the spectrum cannot have much to do with the excitation temperature or degree of ionization of the cepheid atmosphere. It is important that spectrograms of the highest possible dispersions of representative cepheids of different period be compared with normal nonvariable stars.

Results of the fundamental Struve paper (1944) confirmed and extended by Code (1947) are [38, pp. 372-373]:

1. At minimum light, the spectra of cepheids are nearly normal and, as a function of period, a systematic increase in luminosity from class II to class Ia. Most cepheids, i. e., those with periods between 4 and 25 days, belong to luminosity class Ib.
2. With increasing period, the spectra at minimum are of later type.
3. The spectral types at maximum light are nearly the same, ranging from F5 to F8, with increasing period.
4. The spectra at maximum light (or near before) are, however, not normal. The types are based primarily on the appearance of the G band and the ratios of intensities of Fe I and Fe II lines. The H lines are much too strong, the Ti II lines are conspicuously enhanced, and the Fe II lines are slightly strengthened in comparison with MKK standards in the range F5 Ib to F8 Ib. These peculiarities are almost absent in the cepheids of short period ( $\sim 5$  days) but become increasingly conspicuous with increasing period.

"Among the best-studied cepheids of long period ( $P > 10^d$ ), during an interval on the rising branch of the light curve, lines of Ti II and H and, to a lesser extent, Sr II and Fe II become conspicuously widened and their equivalent widths are increased. This widening is probably not an ionization or excitation effect because (1) in the metallic lines it is not symmetrical with respect to the time of bluest color (maximum light) and (2) it becomes a genuinely resolved doubling of Ti II in at least one of the stars and a genuine doubling of H-alpha in three of them. At the same time, all the lines in the spectrum become somewhat wider, though their equivalent widths are not conspicuously increased. The widening moves closer to the time of maximum light with increasing period. Moreover, the profiles of the H lines do not suggest the

development of a shell spectrum, since the detailed behavior of the doubling consists of the development of longward-displaced cores and therefore must be associated with collapsing, rather than expanding material" [38, p. 377].

"The physical parameters of the cepheid atmospheres at maximum light, viz.,  $T_{\text{exc}}$ ,  $T_{\text{ion}}$  and  $\log P_e$  are very similar to those of normal F3 Ib to F7 Ib stars, no matter what the period may be. Thus the color of cepheids at maximum light ought to be nearly constant as a function of  $P$ " [38, p. 378]. There is no evidence in favor of the hypothesis that cepheids develop expanding or stationary shells at maximum light [20].

The existence of small-scale turbulence as revealed by curve-of-growth studies is a well-known and well-established feature of supergiant stars of late spectral type. The microturbulent velocities for cepheids increase with increasing period, i. e., increasing luminosity. The most likely way of clarifying the situation for a particular line in a given star would be as follows:

1. Calculate from the curve-of-growth parameters the line profile to be expected.
2. Broaden this profile by some suitable function corresponding to rotation, large-scale radial motions, etc.
3. Test, by means of some evolutionary track, the consistency of the derived rotational velocity with the rotational velocity of the main sequence progenitor [38, pp. 378-379].

Among both variable and nonvariable stars there is an increase in line width with increasing luminosity. The spectra of cepheids do not differ greatly from nonvariable nearby supergiants in the H-R diagram. Evolutionary tracks of cepheids in the H-R diagram appear in Figure 12 [38, p. 38].

Only two cepheids have had a complete cyclical analysis. They are  $\delta$  Cephei and  $\eta$  Aql. A discussion of a cyclical analysis of  $\eta$  Aql appears in Stellar Atmospheres, Chapter 9, pp. 389-390.

"A recent study of the Ca II emissions in G, K, and M stars by Wilson and Bappu (1957) is of the greatest interest both for astrophysics and for galactic-structure problems. It is found that over a range of almost 16 magnitudes, a close correlation exists between the width of the Ca II emission and the absolute visual magnitude. The sun is found to fit the relation, and it

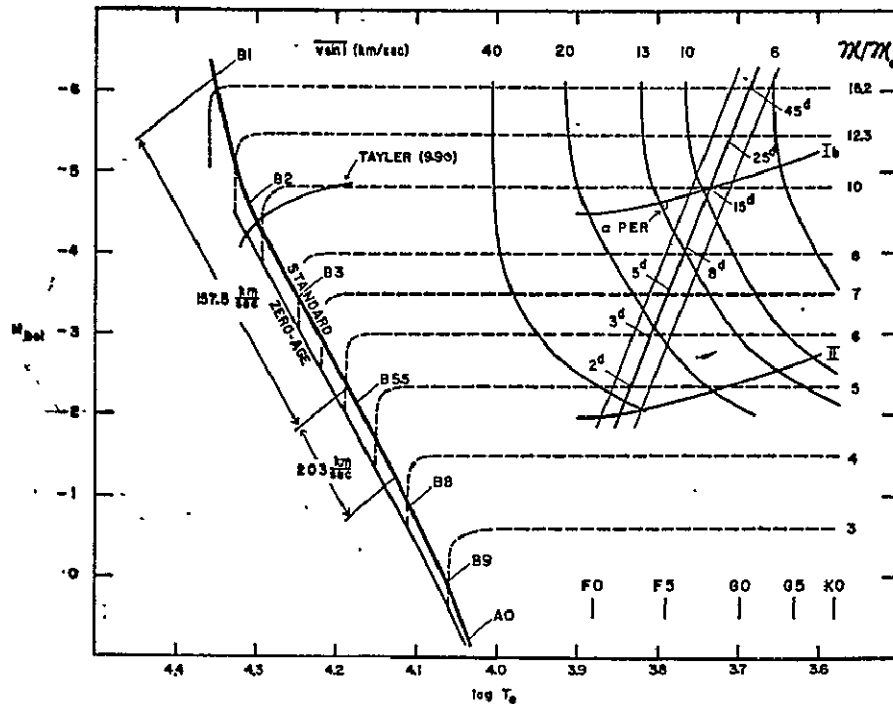


FIGURE 12. EVOLUTIONARY TRACKS IN THE H-R DIAGRAM  
(after Sandage 1958)

therefore seems likely that if an explanation for the phenomenon can be found in the sun, a similar process must work in the stars. Among the supergiants, the emission is wide and, expressed as a velocity, increases from about 100 km/sec at  $M_V = -2$  to 200 km/sec at  $M_V = -6.8$  [38, p. 397]. The Wilson-Bappu correlation is shown in Figure 13 [32, p. 351].

"Aside from the widths themselves, the emission lines have interesting radial-velocity properties. It is found that many stars, especially those of higher luminosity, have an absorption component (or components) superimposed on the emission; the employment of the usual notation of K2 for the emission and K3 for the non-interstellar absorption will follow. On the average, it turns out that K2 shows a small ( $\sim -1$  to  $-2$  km/sec) outward motion and K3 a similar inward motion when measured with respect to the nearby photospheric lines of Fe I. However, among the M stars of luminosity greater than  $+1.5$ , a large majority have K3 shortward-displaced, a phenomenon probably resulting from secular mass loss in the manner of alpha Her (Deutsch 1956)" [38, p. 397].

"The Wilson-Bappu results are particularly remarkable because they imply that the width of K is determined solely by the absolute visual magnitude and not at all by the photospheric temperature or surface gravity" [38, p. 397].



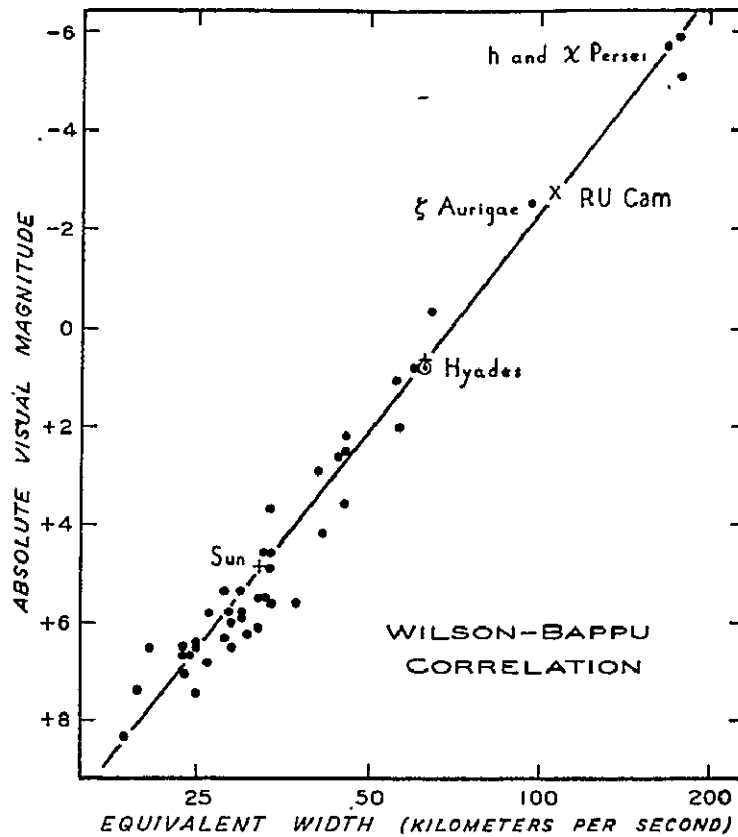


FIGURE 13. WILSON-BAPPU CORRELATION

There are five points to consider concerning the Ca II emission in the spectra of the classical cepheid variables [38, p. 399].

1. In cepheids, the emission is transitory; it appears shortly after minimum light and lasts for a longer fraction of the period with increasing period. In no case, however, does it last longer than about 0.4 of the period.
2. No emission is found in cepheids with periods under four days. This may reflect nothing more than the fact that such stars do not become later than GO; Ca II emission is rarely observed in nonvariable stars.
3. It is broad and hazy on the shortward side and sharp on the longward; however, at phases when the emission appears, the K<sub>3</sub> core is usually displaced greatly longward, and the shape of the resulting profile may reflect only a differential radial-velocity effect.

4. In the spectra of only two cepheids is there a component of K2 longward of the K3 core (or cores). The total emission feature in  $\chi$  Cyg is found to be displaced shortward of the nearby Fe I lines by 15 km/sec; in  $\zeta$  Gem the displacement is about zero. However, the probable error of making such a measurement is large ( $\pm 10$  km/sec) because of the hazy boundaries of the feature.

5. The peak intensity of H often exceeds that of K.

"So far as the photosphere is concerned, the classical cepheids have spectra that, in detail, are virtually identical with those of the nonvariable supergiants at all phases of the variation. The properties of the emission lines, however, are puzzling and may or may not arise from an activity common to the two kinds of stars. Another kind of connection between the cepheids and nearby stars in the H-R diagram is the suggestion by Abt (1957) that all stars in the brighter regions of the H-R diagram are, in fact, variable. Radial-velocity variations are found in virtually all stars of class I, and, while these are often quite small and only quasi-regular, there seems to be a  $P \sqrt{\rho/\rho_0} = Q$  relation. The change in line shapes and wavelengths indicates that pulsation is a likely cause of the variability rather than some fluctuation of turbulence. The values of  $Q$  found by Abt turn out to agree within a factor of 2 with those of the classical cepheids" [38, pp. 403-404]. See Figure 14 [38, p. 404].

"The results might mean, for example, that the cepheid phenomenon is a resonance effect such as one has in the classical problem of the forced oscillations of a dissipative system. As the star evolves across the H-R diagram, it tries to pulsate, but resonance conditions set in a fixed narrow region. One observable quantity about cepheids, viz., the secular period changes, may tell us something about the rate of evolution. According to Sandage (1958), a star crossing the region of cepheid instability will change its color by 0.2 magnitude (B-V) and its period by about a factor of 2" [38, pp. 404-405].

The six-color photometry [4] of Stebbins and Whitford is a method, that is actually a transition to the branch of "spectral photometry," whose object is the study of the energy distribution over the whole range of the spectrum. Light curves in six-colors have been taken of Alpha Ursae Minoris,  $\delta$  Cephei, and  $\eta$  Aquilae. These light curves at the six different wavelengths are discussed below.

"Ever since the light variation of Polaris was established by E. Hertzsprung in 1911, this star has stood out at one end of the sequence of

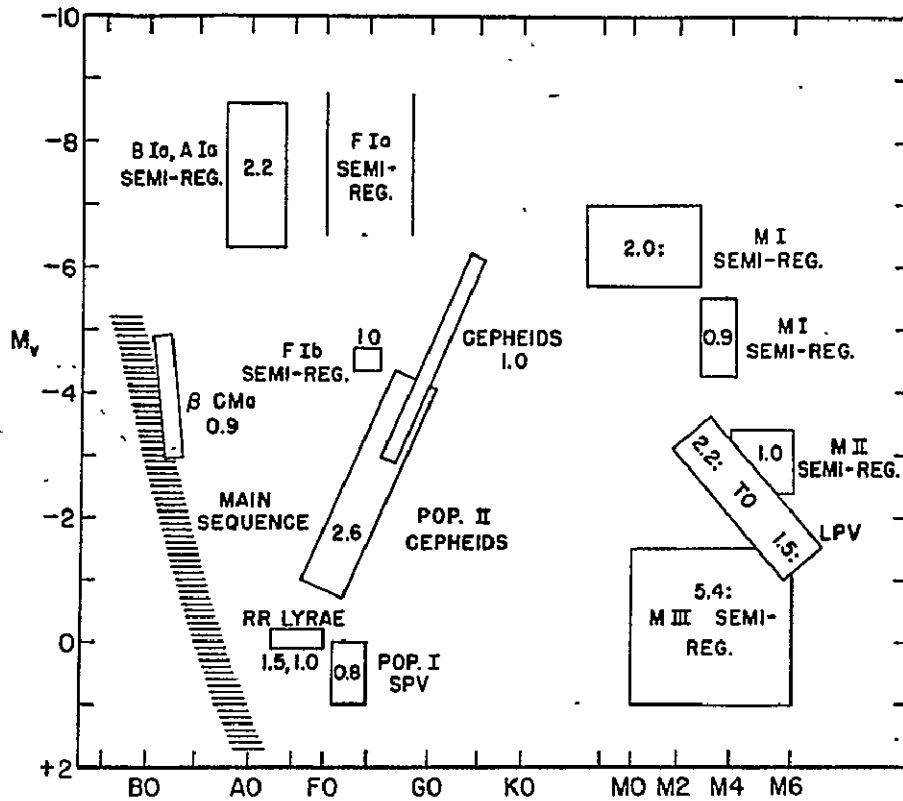


FIGURE 14. Q CORRELATIONS

cepheid variables as having the smallest amplitude of change. Compared with a photographic range of 0.17 magnitude for Polaris, no other star in H. Schneller's catalogue for 1941 is clearly of the same type, with a variation of less than 0.30 magnitude" [39].

Polaris seems to be a true cepheid variable of the  $\delta$  Cephei or  $\zeta$  Geminorum type, repeating its light and spectral changes in the period of 3.96 days continuously for years.

"The amplitudes of the light variation of Polaris in the period of 3.96 days are found to range from 0.166 magnitude at  $3530 \text{ \AA}$  to 0.036 magnitude at  $10300 \text{ \AA}$  in the simple sine curves which characterize the changes of the star. The variation at each wavelength is close to  $1/9$  of the corresponding variation of  $\delta$  Cephei. The inferred variation of spectral type of Polaris is from cF6 to cF7. The period of the variation has become longer than predicted by any of the formulae derived in the past" [39]. The light curves of Polaris in six colors are depicted in Figure 15 [39, p. 111].

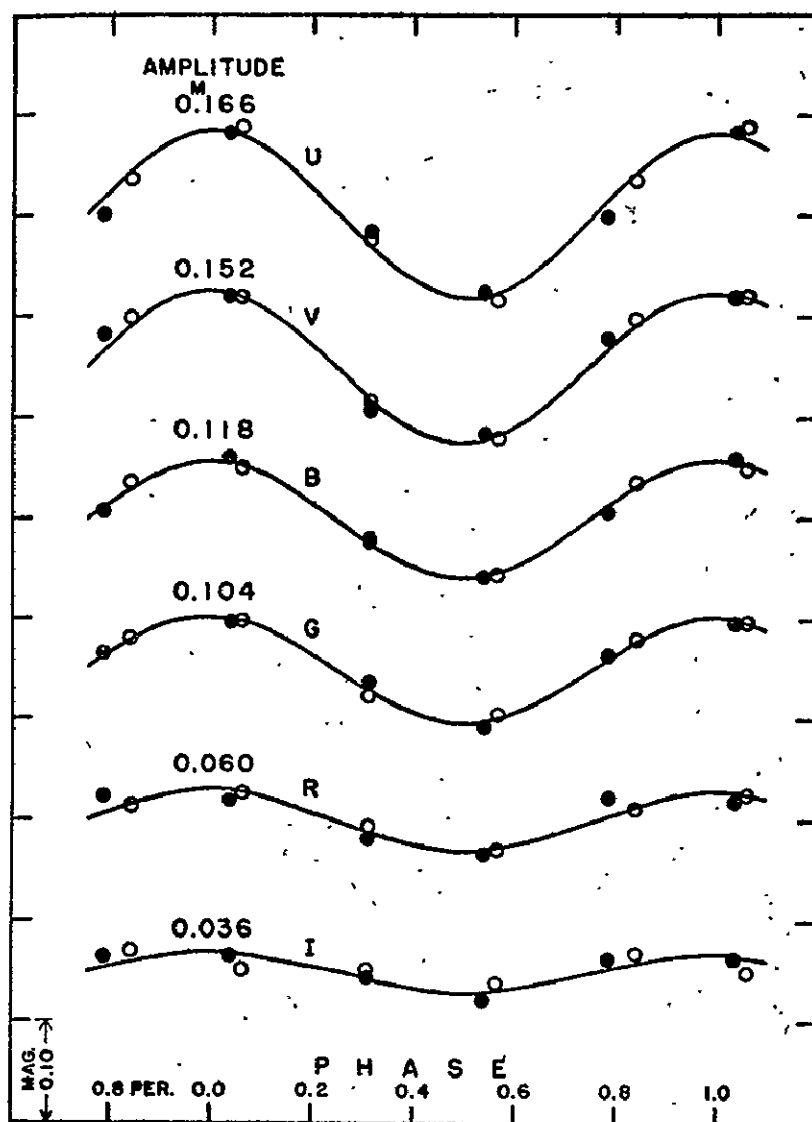


FIGURE 15. LIGHT CURVES OF POLARIS

"Light curves of the variable star  $\delta$  Cephei have been obtained in six colors, ranging from the ultraviolet,  $3530 \text{ \AA}$  to the infrared,  $10\,300 \text{ \AA}$ . These curves are all smooth, without secondary fluctuations, and are repeated accurately over years at a time. The amplitude of variation ranges from  $1.48^M$  in the ultraviolet to  $0.43^M$  in the infrared, the ratio of these extremes being  $1.48/0.43 = 3.4$ . The period of variation has not shortened so much as was predicted from earlier observations. The times of maximum and minimum light are progressively later with longer wavelengths, the displacement of phase

amounting to 0.05 period, or 0.27 day, between 3530 Å and 10 300 Å. The colors and spectrum of  $\delta$  Cephei at maximum light match closely the colors and spectrum of supergiant stars of type cF4, while at minimum the match is with type cG2. Any complete theory of cepheid variation must take into account the displacement of phase with different wavelengths, indicating pulsations in the atmosphere as well as in the radius of the star" [40].

W. Becker has derived an empirical relation between color temperature and radiation temperature. For any spectral region the radiation temperature is defined as the temperature of a black body of the same size as the star in question that would emit the same total energy in that spectral region. When the radiation temperature, which has a much smaller amplitude of variation than the color temperature, is combined with a light curve at any phase, the resulting radius is in good agreement with the relative radius from the velocity curve. Moreover, the combined data of radii and surface brightness of different stars, standardized by similar data for the sun, give absolute magnitudes in excellent agreement with the period-luminosity curve for cepheid variables. Light curves in six colors for  $\delta$  Cephei are seen in Figure 16 [40, p. 49].

"Light-curves of the variable star  $\eta$  Aquilae have been obtained in six colors from the ultraviolet at 3530 Å to the infrared at 10 300 Å. The variations in all colors show the well-known hump in the light curve about half way down in the decreasing phase, with indications of other secondary fluctuations. The amplitudes of variation range from 1.45 magnitudes in the ultraviolet to 0.37 in the infrared. The times of maximum and minimum come progressively later in the longer wavelengths, the displacement of phase being 0.05 period between 3530 Å and 10 300 Å, the same retardation that was found in  $\delta$  Cephei. The colors of  $\eta$  Aquilae at maximum nearly match those of supergiant stars of spectrum cF7; at minimum they approximate cG3; but the variable is not quite a typical star at either phase. The mean radius is found from the light and color curves, combined with an integrated velocity curve, to be  $R = 4.7 \times 10^7$  km = 68  $R_{\odot}$ " [41]. The period of this variable, 7.<sup>d</sup>1767, seems to be lengthening. Light curves in six colors for  $\eta$  Aquilae are shown in Figure 17 [41, p. 295].

The cooler stars have most of their light concentrated in the red. As soon as molecular bands make their appearance, these bands can produce large local distortions in the energy curves [42].

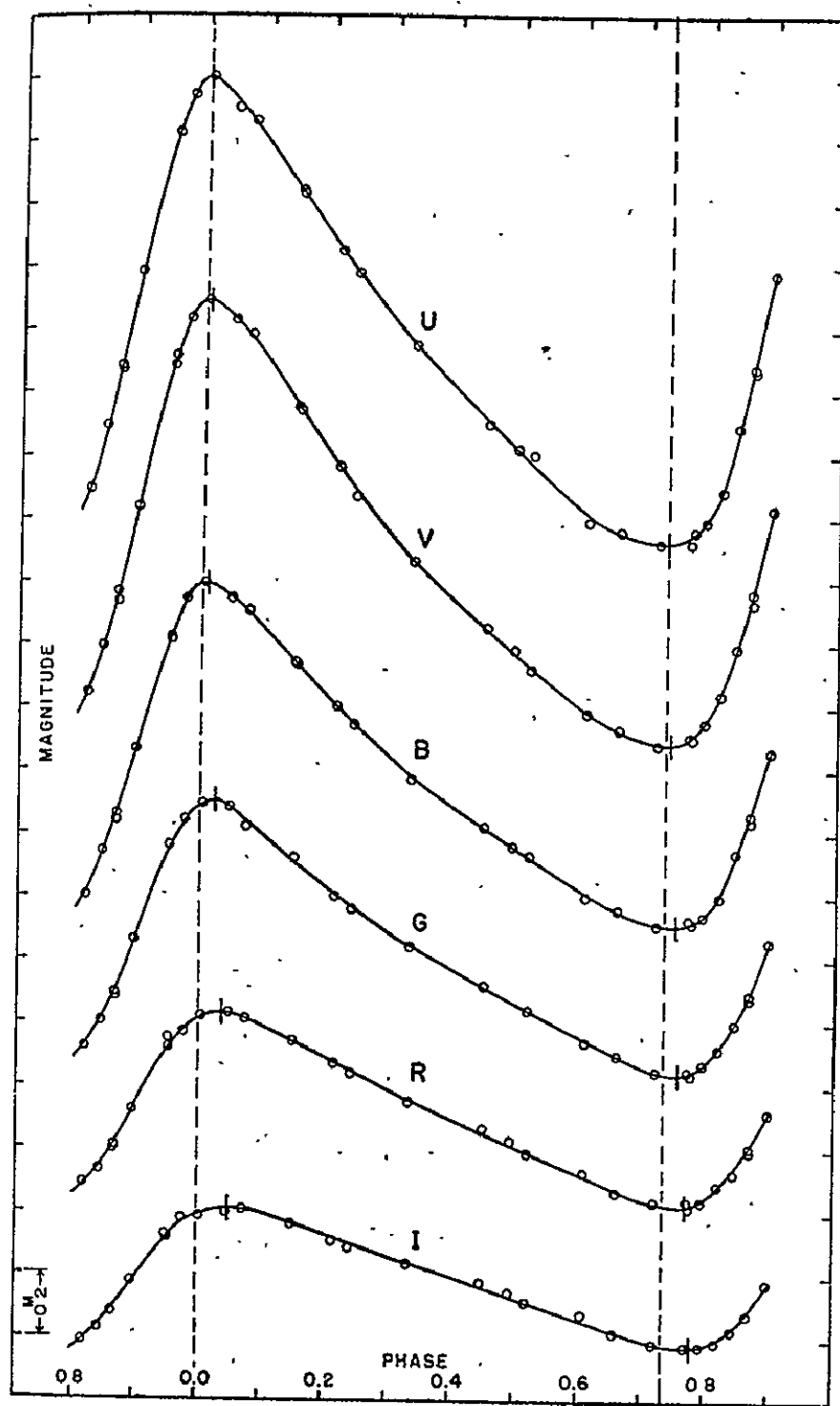


FIGURE 16. LIGHT CURVES OF  $\delta$  CEPHEI

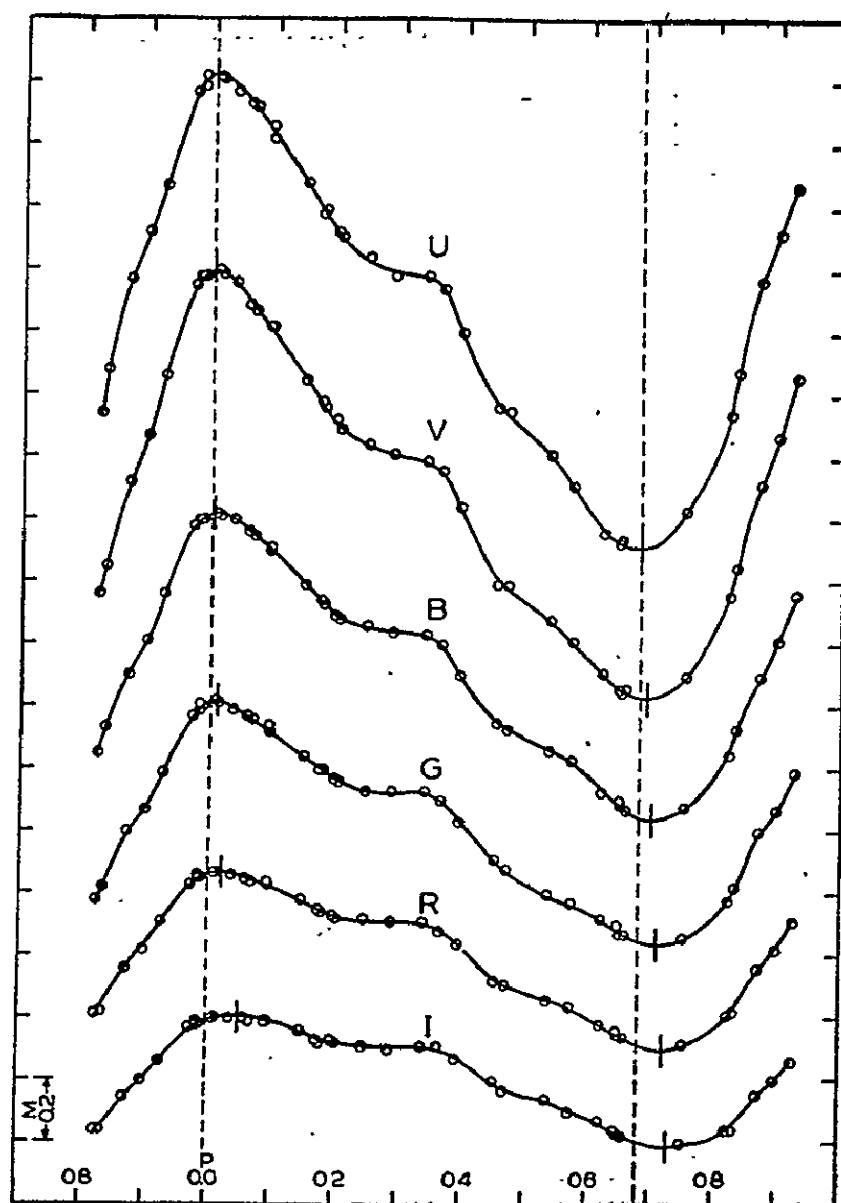


FIGURE 17. LIGHT CURVES OF  $\eta$  AQUILAE

All carbon stars are subject to heavy localized absorption by the well-known Swan bands of  $C_2$  near  $\lambda\lambda$  4750, 5165, and 5635, and by the red system of CN throughout the near infrared [42, p. 7]. "The first astronomical spectrograms covering the long wavelengths were actually taken early in the present century by V. M. Slipher and others, but it was the paper by Merrill in 1934, with its excellent reproduction of the  $\lambda\lambda$  8220 to 8800 region in several stellar spectra, that showed astrophysicists how much could be done in the photographic infrared" [42, p. 8].

"The spectroscopic value of the red end of the spectrum is determined by the absorption features — atomic lines and molecular bands — found there" [42, p. 8]. The strong atomic lines are much less crowded here than in the blue. It is generally true that the elements that are abundant in the cooler stars just do not have as many strong lines per unit  $\lambda$  in the red as in the blue [42, pp. 8-9]. "One disadvantage of the red region must be admitted: absorption in the earth's atmosphere by the terrestrially abundant molecules  $O_2$  and  $H_2O$  almost blot out the stellar features through considerable stretches of spectrum" [42, p. 15]. This will be covered again in a later section.

## THE INTERSTELLAR MEDIUM AND DISTANCE MODULATION

Most of the interstellar material is found between the stars in the spiral arms of our own and other galaxies. The gas and dust is not homogeneous, but tends to be denser in some areas than in others, giving rise to a patchy, irregular distribution. About 10 percent of the total mass of the Milky Way system consists of interstellar matter [4, p. 121].

"In 1935, Lindblad made the initial investigation of the possibility of grain growth by condensation from the interstellar gas. This line of investigation was pursued further by Oort and van de Hulst and has become, with various modifications, the basis for many of the present theories of the formation of interstellar grains" [43].

"Interstellar grains are solid particles — or at least tightly bound agglomerations of matter — larger than molecules but smaller than about  $10^{-4}$  cm. The grains are very effective in blocking the light from distant stars. This extinction must always be considered in determining accurate spatial distributions of stars and external galaxies. By inference from interstellar polarization, you can observe large-scale features of the galactic magnetic fields. The temperature of the gas is moderated by inelastic collisions of atoms with the grains. Undoubtedly molecule formation occurs on the surface of the grains and it may well be that grains play a role in certain stages of the formation of stars. Therefore, the three important activities of the grains are (a) the negative one of extinction, (b) the positive one of tracer of physical conditions in the interstellar medium, and (c) physical interactions with the other components of the interstellar medium" [43, p. 267].



As early as 1917, Halm made observations which, if correctly interpreted, indicate that starlight may be reddened by extinction in passing through space according to a  $\lambda^{-1}$  law [43]. However, it was Trumpler's work in 1930 on the use of color excesses which provided the first definitive proof that interstellar reddening is characteristic of the extinction [43]. Attempts to establish a measure of the total extinction on the basis of the reddening were and still are extremely useful in correcting distances for extinction. The distance correction depends on the very basic assumption that the wavelength dependence of the extinction is invariant. However, as long ago as 1937, Baade and Minkowski observed definite deviations from normal reddening in the Trapezium [43]. Furthermore, modern theories and observations indicate that a universal reddening law is not to be expected. About 25 years ago, the fundamental investigations of the details of the wavelength dependence of extinction led to the discovery that over a fairly wide spectral range, the extinction varies approximately inversely as the wavelength (the " $\lambda^{-1}$  law") [43].

"Although it has become generally accepted that the grains are dielectric and largely made up of ice, nevertheless, alternative suggestions have been difficult to exclude on the basis of the observations alone. Metallic particles and graphite particles have been proposed because they appear at first glance to make it easier to explain the rather high degree of stellar polarization. Very small particles (large molecules) consisting of free radicals have been proposed by Platt because of doubts in the theory of the growth process of the dielectric grains" [43].

## Extinction [2, p. 1074; 43, pp. 269-271; 44]

By placing into the path of a light ray an obstacle of sufficient size (i. e., the diameter is larger than the wavelength of light), you can observe total absorption of the light by the particle. By placing a layer of dust of the constitution in interstellar space, a certain part of the light rays emitted by the stars will be absorbed, i. e., the brightness of the star will be dimmed in a certain proportion. This proportion evidently will be the same for every spectral region, and if the absorption is expressed like the intensity (in magnitudes), then the total, or general, absorption coefficient will be the same for every spectral region; i. e., the magnitudes will be changed by the same amount, whether they are visual, photographic, photovisual, photoelectric, etc. This means that total absorption does not change the color index, as it

does not change the spectrum. There is a difference, however, when the particles become smaller than or equal to the wavelength. When this occurs, the components of the light with longer wavelength (the red region) will be diffracted around the corners of the particles, while the components of shorter wavelength will be absorbed. The proportional amount of dimming will not be the same for every spectral region; the blue will be absorbed more strongly than the red; the color of the star will appear reddened. Expressed in magnitudes, the photographic absorption coefficient will become larger than the photovisual one, and the color index will increase on account of the selective absorption. The selective absorption coefficient — the difference between the photographic and photovisual absorption coefficients — is a measure of this absorption. The difference between the apparent (reddened) and the true color index of a star is called its color excess, and this is equal to the product of the distance of the star (in kiloparsecs) and the selective absorption of the region.

An important method by which the spectral characteristics of the interstellar medium can be explored is that which compares the observed colors of two stars, one of which is affected by interstellar extinction and the other is not. In the simplest case, observations are made on two stars of identical spectral type; the differences between the colors of the two stars are interpreted as resulting from the interstellar material. The assumption is made that the intrinsic colors of the two stars of the same spectral type are identical. It is more convenient in practice first to derive the intrinsic colors of stars of the various spectral types from observations of unreddened and little reddened stars. The colors of the reddened stars are then compared with the derived intrinsic colors rather than with an individual star of the same spectral type.

Several investigators, for example, Sharpless (1952) and Johnson and Borgman (1963), have interpreted their data as indicating that the interstellar extinction law is not everywhere the same; in particular, these authors have derived a value of  $R = A_V / E_{B-V}$  (the ratio of total-to selective absorption) in the region of the Orion Trapezium that is approximately double that which is found in other regions of the sky. Examples of interstellar extinction curves that have been found in several regions of the sky are shown in Figure 18 [44, p. 133].

The apparent magnitude  $m(\lambda)$  [this means the apparent magnitude of some wavelength  $\lambda$ ] and the absolute magnitude  $M(\lambda)$  of an unreddened star are related by

$$m(\lambda) - M(\lambda) = 5 \log r/10$$

where  $r$  is the distance in parsecs (pc). If the star is reddened, its apparent

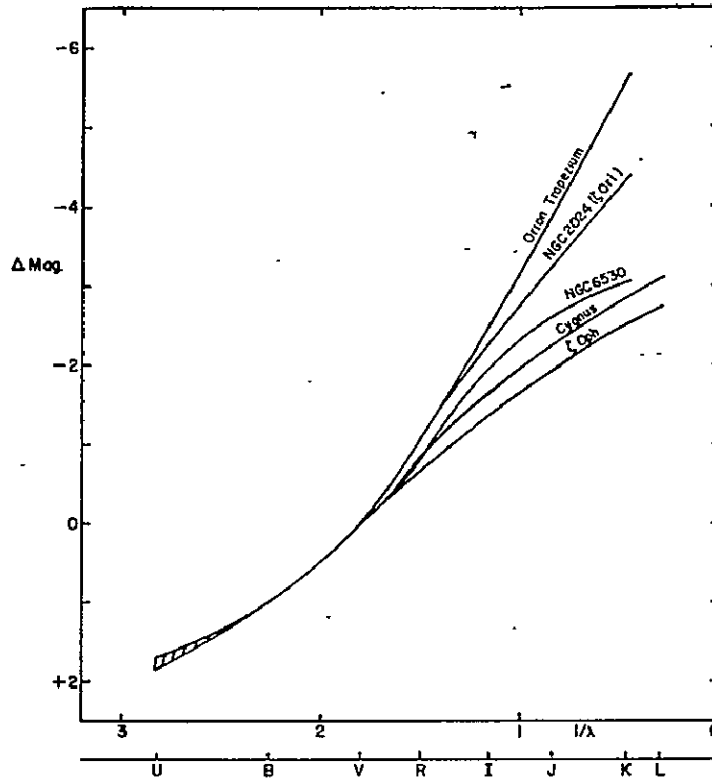


FIGURE 18. THE EXTINCTIONS HAVE BEEN NORMALIZED  
TO  $E_{B-V} - 1.00$

magnitude  $m(\lambda)$  will be fainter, and you can write  $m(\lambda) - M(\lambda) = 5 \log r/10 + A(\lambda)$  where  $A(\lambda)$  is defined as the extinction. Comparing an unreddened with a reddened star (denoted by primes), you get the extinction  $A(\lambda)$  by

$$A(\lambda) = m'(\lambda) - m(\lambda) - M'(\lambda) + M(\lambda) - 5 \log r'/r$$

where, on the right-hand side, the quantity  $M'(\lambda) - M(\lambda) + 5 \log r'/r$  is equal to zero for stars of the same spectral type, luminosity, and distance. The color index (C.I.) of a star is defined as the difference in magnitudes at two wavelengths  $\lambda_i$  and  $\lambda_j$ , where  $\lambda_i < \lambda_j$ . If the light from a star undergoes interstellar extinction which is greater at short than at long wavelengths, the color index is apparently increased. The color excess  $E_{ij}$  of the reddened star is the difference between its apparent and intrinsic color index. Thus

$$E_{ij} = [m'(\lambda_i) - m'(\lambda_j)] - [m(\lambda_i) - m(\lambda_j)] = A(\lambda_i) - A(\lambda_j) .$$

To be able to compare directly the wavelength dependence for different stars, it is customary to normalize the extinction curves. A schematic example of such a normalized Whitford reddening curve is shown in Figure 19 [44, p. 270].

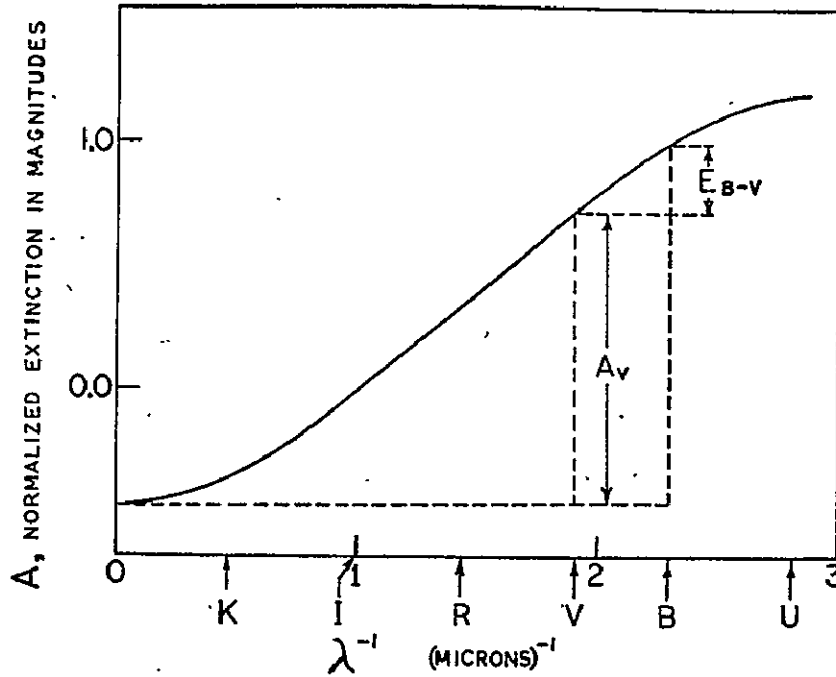


FIGURE 19.  $\lambda^{-1}$  RELATION

One usually normalizes to a standard color excess  $E_{ij} = 1.00$  magnitude where  $\lambda_i^{-1} = 2.4 \mu$  ( $\lambda_i = 4160 \text{ \AA}$ ) and  $\lambda_j^{-1} = 0.99 \mu^{-1}$  ( $\lambda_j = 10\,000 \text{ \AA}$ ). The extinction at each wavelength  $\lambda_k$  is then obtained by finding the ratio  $E_{kj}/E_{ij}$  where  $k$  corresponds to the other colors. This arbitrarily makes the extinction at  $\lambda\,10\,000$  equal to 0.0 magnitudes. In Figure 19, the quantities  $A_v$  and  $A_p$  are the total extinctions in the "photovisual" ( $\lambda\,5290$ ) and the "photographic" ( $\lambda\,4250$ ).

If the wavelength dependence of the normalized extinction curves for all stars were the same and known down to  $\lambda^{-1} = 0$  (zero extinction), then the difference between the extinctions at only two wavelengths would determine uniquely an absolute scale to the extinction curve. It then becomes convenient to define the ratio of total to selective extinction as

$$R = \frac{A(\lambda_1)}{A(\lambda_2) - A(\lambda_1)}$$

where  $R$  is a constant depending on the nature of the extinction curve. Significant deviations have been found from the average extinction curve making  $R$  vary by as much as a factor of two when  $\lambda_1$  and  $\lambda_2$  correspond to the V and B colors as shown in Figure 18. However, there is a sufficient degree of uniformity of the extinction curves for most stars so that the above definition of the ratio of total to selective extinction serves a useful purpose in correcting distance module for the effect of extinction.

### Polarization [43, p. 271]

For a star to exhibit polarization, interstellar absorption is a necessary but not a sufficient condition. The polarization is defined by

$$p = \frac{I_1 - I_2}{I_1 + I_2}$$

where  $I_1$  is the intensity in the plane of polarization and  $I_2$  is the intensity in the perpendicular plane. Expressed as a magnitude difference, the polarization is given by

$$\Delta m_p = 2.50 \log (I_1/I_2) = 2.1717 p \quad p \ll 1.$$

"Polarization is found in the radiation of all types of stars, either single or multiple, early spectral type or late, main sequence or supergiants, intrinsic variables or nonvariables. The maximum polarization is found in stars near the galactic plane and for stars with large interstellar absorption. Further, the plane of vibration of the observed polarization is associated with the plane of the galaxy" [2, p. 1083].

At present, polarization measurements provide the only access to the observational characteristics of the magnetic fields which must play an important role in the structure of the galaxy. Some problems [2, pp. 1090-1091] that may prove significant in polarization measurements are:

1. The variation in polarization with wavelength. To a first approximation, polarization is independent of wavelength. In the infrared, the polarization begins to decrease where at  $\lambda$  8500 Å the polarization has decreased by approximately 20 percent.

2. Polarization of nearby stars with high precision.

3. Observation of double stars and especially star clusters to determine fine structure of the magnetic fields.

4. Observations of polarization, color, magnitude, and spectral type of stars at all galactic longitudes.

The galactic distance scale is sensitive to the reddening correction; four methods of approach to the problem of defining the intrinsic colors of the cepheids follow [45]:

1. One may assume that galactic cepheids and cepheids in the Magellanic Clouds have the same intrinsic colors.

2. One may assume that variable and nonvariable supergiants of luminosity class Ib of the same spectral type have the same intrinsic colors.

3. An estimate of the interstellar reddening for nearby cepheids may be made by determining the reddening as a function of distance from color excesses of B stars in fields containing cepheids.

4. An estimate of the interstellar reddening may be made by applying the correlation between reddening and polarization for O and B stars to the observed polarizations of cepheids.

There also exists a fifth method, the ad hoc method, where the bluest stars of a given class are used to define the intrinsic colors.

## Distance Modulation

The determination of distances to cepheid variables is a very difficult observational problem. In the first place, there is not a single cepheid near enough so that its trigonometric parallax can be measured; therefore, statistical methods involving the proper motions and radial velocities of these stars must

be employed. In the second place, most of the cepheid variables in our galaxy lie close to the plane of the galactic system, where clouds of interstellar dust heavily obscure their light. Corrections must be applied, therefore, to the measured apparent magnitudes of these cepheids; these corrections are difficult to determine accurately and are thus rather uncertain. This correction factor was not taken into account in Shapley's early work. On the average, it is found that starlight is weakened by about 1 or 2 magnitudes for each kiloparsec of distance it traverses [4, p. 117]. The actual amount of absorption can differ appreciably from star to star; it depends both on the distance and on the direction of the star.

In 1856, Pogson defined the magnitude-light ratio to be exactly

$$\left(\sqrt[5]{100}\right)^{m_2 - m_1} = \ell_1 / \ell_2,$$

where  $\ell_1 / \ell_2$  is the ratio of light intensities and  $m_2 - m_1$  is the change in magnitude associated with the light intensities. Therefore, for a change in magnitude of 1, the corresponding light ratio would be  $\sqrt[5]{100} \sim 2.512$ . It is also seen that

$$\left(\sqrt[5]{100}\right)^{m_2 - m_1} = \ell_1 / \ell_2 \rightarrow m_2 - m_1 = 2.5 \log (\ell_1 / \ell_2).$$

Using absolutes (magnitude), this reduces into the distance formula that is readily known,  $m - M = 5 \log r/10$ , where  $m$  is the apparent magnitude,  $M$  is the absolute magnitude, and  $r$  is the distance to the star in parsecs (pc). This formula will allow accurate distance determination for all unreddened stars. But most stars are reddened to some extent, thereby producing a correction factor to the distance formula, i. e. ,

$$m - M = 5 \log r/10 + A(\lambda) .$$

This correction factor has been shown to be correlated to color excess, thereby allowing an estimation to be made. The total absorption [1, p. 458], in visual magnitudes, is found empirically to be about three times the  $m_{pg} - m_{pv}$  color excess. Therefore, the distance modulus of a star dimmed by interstellar obscuration is given by

$$m_v - M_v = 5 \log r/10 + 3 \text{ CE}$$

where  $m_v$  and  $M_v$  are its apparent and absolute photovisual magnitudes,  $r$  its distance in parsecs, and CE its photographic-minus-photovisual color excess.

## ASTRONOMICAL INFRARED MEASUREMENTS

Infrared astronomy had its origin in observations made by Sir William Herschel in 1800. By using differently colored darkened glasses (filters) he noted "sensations of heat" through some with but little light, and others much light, but little heat. "This prompted Herschel to conduct a famous experiment in which he used a prism to separate sunlight into its various colors and compared the temperature at different points in the spectrum by means of a glass thermometer whose mercury bulb had been blackened with ink. Two other thermometers located out of the path of the sun's rays served as controls. Herschel found that the sunlit thermometer showed a steady increase in temperature as he moved it from the violet end of the spectrum to the red end, and that the temperature remained high for some distance beyond the visible red" [46]. Herschel subsequently demonstrated that this "infrared radiation" obeyed the same laws of reflection and refraction that apply to visible light and is produced by many sources.

"Today, the term 'infrared radiation' is commonly used to describe electromagnetic radiation whose wavelength lies between  $0.75 \mu$ , which is just beyond the red end of the visible spectrum, and  $3000 \mu$ , the beginning of the region that can be detected by microwave radio wave techniques" [46].

The region between  $22 \mu$  and  $1000 \mu$  is still almost entirely unexplored, however, largely because little radiation in this region can penetrate the earth's atmosphere. "For convenience, infrared astronomers have labeled portions of the infrared region according to the mode of detection or to the bandwidth of particular 'windows' in the atmosphere. Thus, the region from  $0.75 \mu$  to  $1.2 \mu$  is called the photographic infrared, because photographic emulsions still respond to radiation of such wavelengths. The first photograph in the infrared was made by Sir John Herschel in 1840, before suitable silver-salt emulsions were available. He recorded a portion of the infrared spectrum of the sun by exposing paper that had been soaked with alcohol containing particles of carbon black; the heat of the infrared radiation evaporated the alcohol in such a way as to leave a detectable image" [46, p. 20-21].



"All astronomical observations beyond  $1.2\ \mu$  are currently made with one-dimensional detectors, detectors that respond only to the radiant energy collected at a single point in the image. The region from  $1.2\ \mu$  to  $5.2\ \mu$  is usually called the near infrared. In the region from  $5.2$  to  $8\ \mu$  the earth's atmosphere is completely opaque. There is a window between  $8$  and  $14\ \mu$  (the long wavelength infrared); another window exists between  $17$  and  $22\ \mu$ . No important astronomical measurements have yet been made in the region between  $22$  and  $1000\ \mu$ " [46, p. 21]. The first of the one-dimensional infrared detectors was the thermocouple which consists of a junction of two dissimilar metals; heating of the junction generates a small flow of electric current. A parallel array of thermocouples is known as a thermopile; the glass lenses in a refracting telescope will not transmit radiation much beyond  $1.5\ \mu$ .

"In 1880, Samuel Pierpont Langley invented another one-dimensional detector: the bolometer, a device that measures the increase in electrical resistance that results when a blackened metal foil is heated by absorbing radiation" [46, p. 23].

"W. W. Coblentz measured the near infrared emission of stars for the first time. In the late 1920's, Edison Pettit and Seth B. Nicholson introduced the concept of the stellar bolometric correction, a correction applied to estimates of the energy of stars that is based on infrared measurements out to  $14\ \mu$ " [46, p. 23]. Arthur Adel pushed infrared measurements to  $22\ \mu$ . Next came the Golay cell, [46, p. 23] invented by Marcel J. L. Golay in 1942; the heating of a confined gas creates a pressure that slightly distorts a mirror against which a beam of light is directed. The infrared emission of the earth's atmosphere is particularly strong in the  $8$  to  $14\ \mu$  region of the spectrum.

The long-sought goal of producing a cooled and shielded thermal detector was attained in 1960 by Frank J. Low [46, p. 24] at the firm of Texas Instruments, Incorporated. Low developed a bolometer that operates at a few degrees Kelvin. The bolometer is at the bottom of a cylindrical metal shield that is refrigerated by liquid nitrogen. Thus, few infrared photons can reach the detector except those defined by the cylinder's field of view. With his cooled detector, Low was successful in observing the  $8$  to  $14\ \mu$  radiation from Saturn for the first time.

Quantum infrared detectors or photoconductive detectors have the ability to absorb individual infrared photons and to release a unit of electric charge for each photon absorbed. The conductivity of the crystal depends on the number of charge carriers released; this conductivity can be used as a measure of infrared flux [46, p. 24].

A standard arrangement for using sensitive detectors is depicted in Figure 20 [46, p. 25]. The detector is exposed alternately to two beams; one beam consists of radiation from the celestial object and the surrounding sky, while the other is of the sky radiation alone. A rotating chopper blade determines which beam reaches the detector. If the infrared flux of the two beams is different, the detector will produce a fluctuating signal proportional to the differences in radiation energy in the two beams. The signal is rectified to yield direct current voltage, which is recorded. Schematic diagrams showing the arrangement of the apparatus and the temperatures of the various parts are shown in Figures 21 [46, p. 25] and 22 [47]. In the 22 to 1000  $\mu$  region, the only source of opacity in the earth's atmosphere arises from water vapor, the absorption bands of which are broadened by atmospheric pressure.

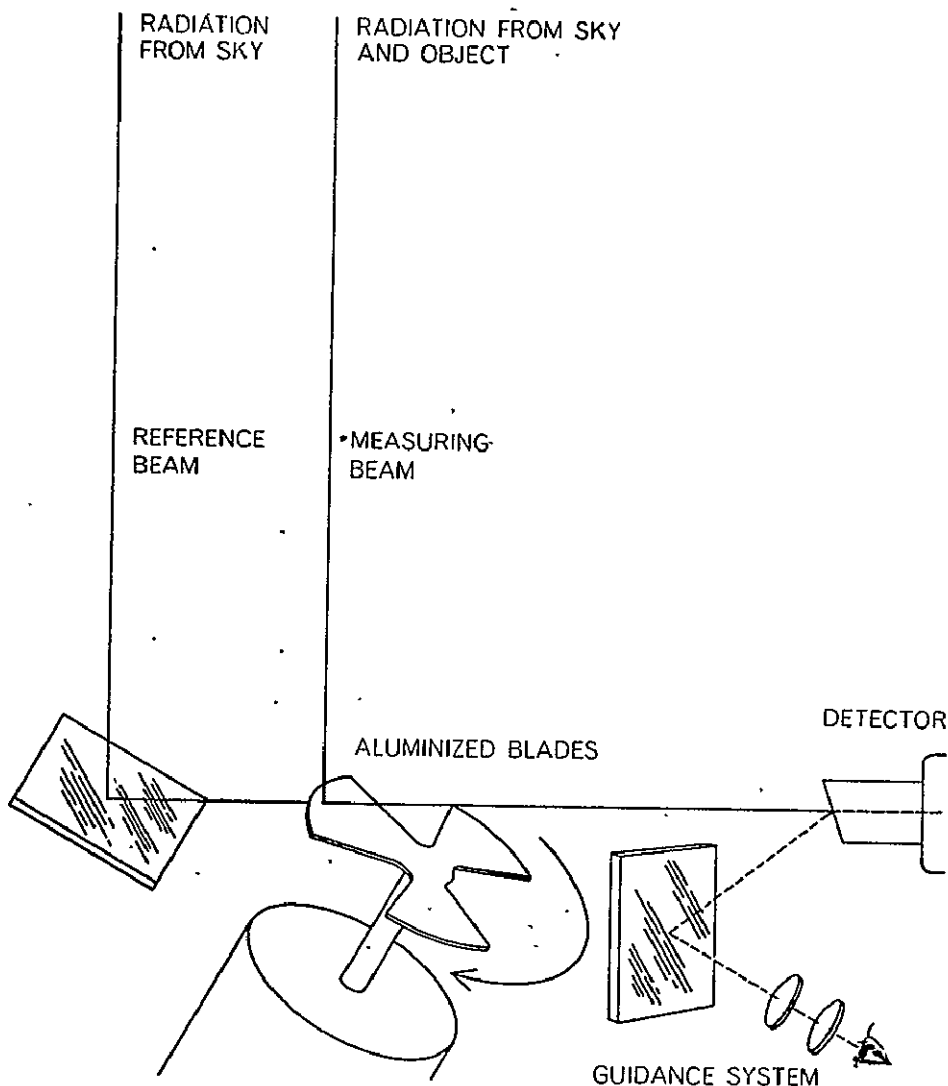


FIGURE 20. INFRARED DETECTOR OPTICAL SYSTEM

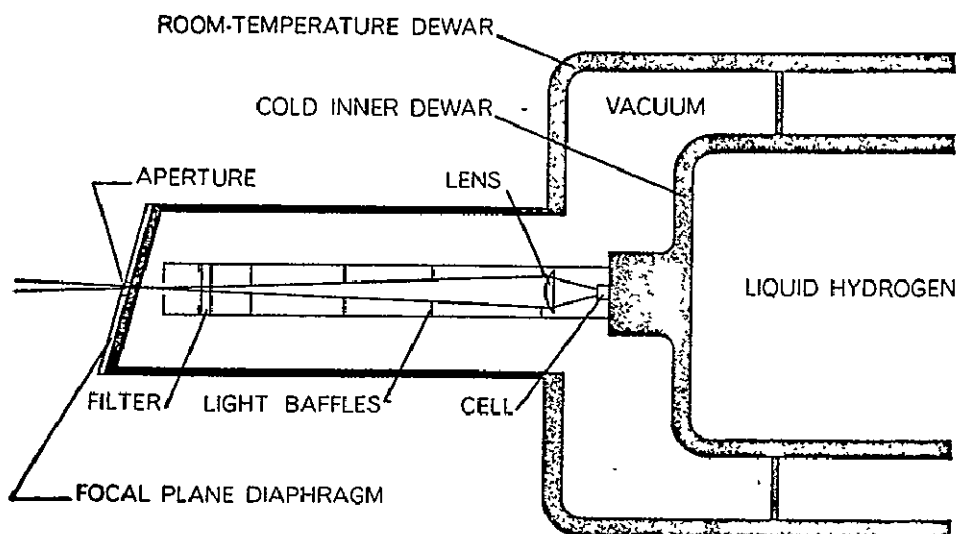


FIGURE 21. A COOLED AND SHIELDED DETECTOR

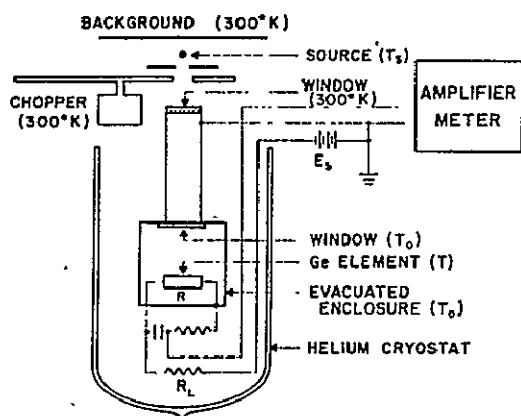


FIGURE 22. A COOLED DETECTOR SYSTEM

The main absorption (in the atmosphere) of energy is caused by polyatomic constituents [2, p. 863]. These gases possess strong vibration and rotation absorption bands in the infrared and nearly all the energy of temperature radiation from a source at ordinary atmospheric temperatures lies in the range of wavelengths between  $3 \mu$  and  $60 \mu$ . "The fundamental bands of water vapor and carbon dioxide are so strong, and their overtone and combination bands so numerous, that few regions in the infrared atmospheric spectrum are completely free from their effects" [2, p. 865].

"The most striking features in a low dispersion spectrum, say  $1 \mu$ , are the  $\phi$ ,  $\Psi$ ,  $\Omega$ ,  $\chi$  bands of  $H_2O$ , the  $4.3 \mu$   $CO_2$  band, the  $6.3 \mu$   $H_2O$  band, the  $9.6 \mu$   $O_3$  band, and the great rotation band of water vapor which, at sea level, absorbs completely solar radiation beyond about  $13.5 \mu$ , although Adel and Migeotte and Neven, at high altitude stations, have detected a small amount of energy between  $16 \mu$  and  $22 \mu$ " [2, p. 865].

Weaker features occurring in the transparent regions include bands at  $4.5 \mu$  and  $3.9 \mu$  caused by  $N_2O$  and a band at  $3.7 \mu$  caused by DHO [2, p. 865]. "Methane has two fundamental infrared absorption bands at  $3.3 \mu$  ( $\nu_3$ ) and  $7.7 \mu$  ( $\nu_4$ ), respectively. Migeotte first detected the  $\nu_3$  band in the solar spectrum at Columbus, a low altitude station, and also identified some lines belonging to the  $\nu_4$  band in a spectrum of the  $7.7 \mu$  region published by Adel in 1941" [2, p. 868]. The fundamental CO band is at  $4.6 \mu$ . The main source of supply of these bands is probably at the ground [2, p. 869].

## CEPHEID RESEARCH

The best-known examples of pulsating stars are cepheids. They have been shown to be stars of high intrinsic luminosity and have been shown to have a light variation characteristic unto itself. The absolute magnitude is related to the period of variation according to the period-luminosity relation. Also, the majority of the known cepheids belong to Baade's population I group and, therefore, are strongly concentrated toward the galactic plane. These characteristic properties make the cepheids a very suitable group of variable stars for an investigation of interstellar absorption near the galactic plane, and also for an investigation of spiral structure in the neighborhood of the sun up to distances measuring several kiloparsecs. "The combination of photometric data with radial velocities and proper motions will provide valuable information concerning the dynamical properties of the galactic system in a wide region around the sun" [48]. Therefore, it has been generally recognized for several years that accurate photometry of cepheids is of fundamental importance.

Earthbound telescopes are confined in that they can be used only for detecting the particular wavelengths that pass through the several known "windows" in the atmosphere. It is possible that several other unknown infrared windows exist that can be used in infrared research; only experimentation can tell. In other words, infrared techniques can be applied to determine atmospheric absorption and transmission windows, thereby allowing further cepheid research in the infrared.

When discussing infrared radiation, we are naturally led to asking certain basic questions [49], many of which apply to stellar evolution and the expansion of the H-R diagram. Can one detect the infrared radiation of "proto-stars," condensing clouds of matter that have not yet become hot enough to emit visible light? Which of the stars that emit light have cool, invisible companions, or shells (expanding atmospheres), or clouds of surrounding gas or dust?

What will infrared observations reveal about the cloud-obscured nucleus of the galaxy? What of determining the surface temperature of the moon, the planets (planetary atmospheres), asteroids, or the stars? What clues can come from the infrared observation of the luminous objects throughout the universe?

With the use of satellites [50], one could get above the obscuring atmosphere of the earth and observe all radiation (bolometric). This is of crucial importance for studying the absolute energy distributions of stars and is of crucial importance for establishing the bolometric correction (B.C.) scale. The ground based astronomy [50] could then do needed high resolution spectrophotometric measurements and high dispersion spectroscopic data for determining the molecular abundances and rotational temperatures.

The 6-color photometry can be expanded further into the far infrared, giving better pictures of cepheid radiation. Possibly evidence concerning the pulsation theory and evolutionary theory of the cepheids will become available by the use of infrared techniques.

One thing is known, that with the expansion of infrared techniques and instruments, much can be learned concerning cepheids and the universe in general. NASA (especially at MSFC) can become a leader in the new field of research.

## APPENDIX A

### PULSATION THEORY

The pulsation hypothesis was subjected to a penetrating analysis by Eddington. He considered the theory of adiabatic pulsations of a gaseous star with a given density distribution, and solved the equations for pulsations in the lowest mode of a standard model star. He also showed that the observed skewness of the light and velocity curves is in qualitative agreement with what might be expected from the influence of second order terms in the equations.

"Eddington's work on second-order effects has been carried further by several investigators, notably J. Woltjer, Jr. and Miss H. A. Kluyver. These authors have successfully introduced principles of celestial mechanics into pulsation theory. Woltjer, in particular, proposed the hypothesis that the shape and magnitude of the velocity curve is primarily due to resonance coupling between the first and a higher mode of pulsation. Miss Kluyver used the same idea for an explanation of the long period variation of the light curve of the cluster variable RR Lyrae" [30].

"On the simple pulsation theory, where the changes in the star are adiabatic throughout, the luminosity is in phase with surface temperature and the radius variation (maximum luminosity occurring at minimum radius). Actually, luminosity is nearly in phase with the pulsation velocity, and hence lags a quarter period behind the state of highest compression. Similarly, minimum light coincides with the highest velocity of contraction, and so lags a quarter period behind the time of greatest distension of the star" [30].

"Though the simple pulsation hypothesis has been generally accepted, several alternative possibilities, or variants of the hypothesis, have been discussed. Thus, Jeans proposed the hypothesis that cepheids and long-period variables are stars in the process of fission, giving a vivid illustration in the sky of the transition of a rotating star from a configuration of a rotating Jacobian ellipsoid through a pear-shaped figure to a true binary. This idea ran up against the objection that, with the high velocities of rotation implied, the spectral lines would have to be broad and diffuse. This is contrary to fact, since cepheids have very narrow and sharply defined lines, the so called c-characteristic" [30, p. 8].

"The theory of Hoyle and Lyttleton is akin to the view of Jeans, though differing in details. These authors visualize a cepheid as a very close binary, a contact binary in fact, surrounded by an atmosphere that does not share perceptibly in the rotation of the mass doublet inside it. The fission process is thus assumed to have been virtually accomplished. Since the common atmosphere of the binary does not rotate, the spectral lines will only show pulsational displacements. The objections raised against Jeans' idea of a binary in a process of fission are thus avoided" [30, p. 8].

"Milne attempted to develop a theory of intrinsic stellar variability which was based on energy considerations alone. Pannekoek has proposed a modification of the pulsation hypothesis which will require careful consideration. From an analysis of the curve that connects the intensity of a spectral absorption line with the number of absorbing atoms per unit area of the star (the curve of growth), Pannekoek drew the tentative conclusion that in the atmosphere of  $\delta$  Cephei, the atmospheric density gradient corresponds to a surface gravity 10 times smaller than that computed from estimated mass and radius. This result was assumed to indicate that there is a steady outward flow of matter in the atmosphere, which is superposed on the pulsation. The combined result of the two phenomena will be the throwing off of successive shells of matter from the star. The cepheids would then behave like miniature novae. The same conclusion was arrived at independently by Menzel, and from different considerations. There is, of course, nothing that conflicts with the pulsation hypothesis in such an idea, as a steady outflow of matter is a common feature among the stars. If such a star for some reason starts pulsating, it is to be expected that the steady outflow will be converted into a throwing off of successive shells. But further research would seem to be necessary before accepting this as a fact that plays an essential part in cepheid phenomena" [30, p. 9].

"The Pannekoek-Menzel picture of the cepheid phenomenon goes beyond Schwarzschild emphasis on the running wave characteristic of the atmospheric pulsation of a cepheid. An ordinary running wave does not produce a net transfer of matter unless it develops into a shockwave" [30, p. 9].

"Since for these stars period depends on density, this period-luminosity relation suggests that for a star of given luminosity in the range where cepheids are found, there is a definite density at which the star starts pulsating" [30, p. 13].

"Just what condition is brought about at this particular density is a matter of speculation. One may think of a change-over from one type of energy generation to another with a higher temperature sensitivity than a quiescent

star can tolerate. Or, one may think, as was suggested by Gamov and Teller, that pulsations start when a star has consumed a particular kind of nuclear fuel and starts a career of gravitational contraction. Eddington, on the other hand, held the view that pulsations are induced because the ability of the star to dissipate energy of wave motion suffers a sudden diminution. As a cause of such a diminished dissipation Eddington referred to the building up of the hydrogen ionization zone in the subphotospheric region" [30, p. 13].

"Although the cause of the pulsation is not clearly understood, it is supposed to be a transitory disturbance in the life of a star as it evolves across the top of the color-magnitude diagram. The region of instability in the color-absolute visual magnitude diagram, according to Sandage, is a strip of width about  $0.2^M$  in color index. This strip extends from  $M = +4$ ,  $B-V = 0.0$ , to  $M = 6$ ,  $B-V = 0.7$ ; it includes all cepheid and RR Lyrae variable stars" [17, p. 389].

A remarkable confirmation of the pulsation theory was first observed independently by Otto Struve at McDonald Observatory and R. F. Sandford at Mt. Wilson in the spectrum of RR Lyrae.

It is therefore seen that there is a close correlation between the pulsation period and other physical parameters. The longer the period, the later is its spectral type, the larger its radius, its brightness, and its mass, and the smaller its effective temperature. There must exist definite combinations of these physical parameters which tend to render the star unstable.



## APPENDIX B

### EVOLUTIONARY THEORY AND THE H-R DIAGRAM

One of the most important of the many possible evolutionary diagrams is the Hertzsprung-Russell Diagram, which is based on independent work done by Ejnas Hertzsprung of Denmark and Harry Norris Russell of the United States. As early as 1905, Hertzsprung noticed that nearly all blue stars were intrinsically very bright, but on the other hand there were two kinds of red stars [20]; high-luminosity and low-luminosity. The meaning and implications of the correlation between luminosity and spectral type were first fully understood by Russell. Russell presented his paper in 1913 to the AAS. He concluded the 1913 paper by stating that these facts have a decided bearing upon the problem of stellar evolution. As mentioned, there are three commonly used choices of parameters in constructing an H-R diagram. The first plots made by Hertzsprung in 1911, by Russell in 1913, and by H. D. Curtis in 1922 show absolute visual magnitude versus spectral type [20, pp. 261-262]. The form of diagram more commonly used today gives absolute visual magnitude (V) versus color index (B-V), which is essentially the difference between photographic and visual magnitude. The third version, used mainly in theoretical investigation, is a plot of either the bolometric magnitude or the log of the luminosity versus the log of the effective temperature.

The accompanying H-R diagram was constructed some years ago by the Swedish astronomer W. Gyllenberg, using all stars with known visual absolute magnitudes and spectral types. It is shown in Figure B-1 [20, p. 262]. Note that the main sequence is not a thin line but a broad band with a width of about three magnitudes.

Basically, the H-R diagram is a plot of stellar luminosity against stellar temperature. Hot (blue) stars are plotted toward the left, cool (red) ones to the right; intrinsically luminous stars lie near the top, intrinsically faint ones toward the bottom.

Looking at the features of the H-R diagram, it is seen that the most striking feature of the diagram is that the stars are not distributed over it at random, exhibiting all combinations of absolute magnitude and temperature, but rather cluster into certain parts of the diagram. The majority of stars are aligned along a narrow sequence running from the upper left to the lower right of the diagram. This band of points comprises the main sequence. A

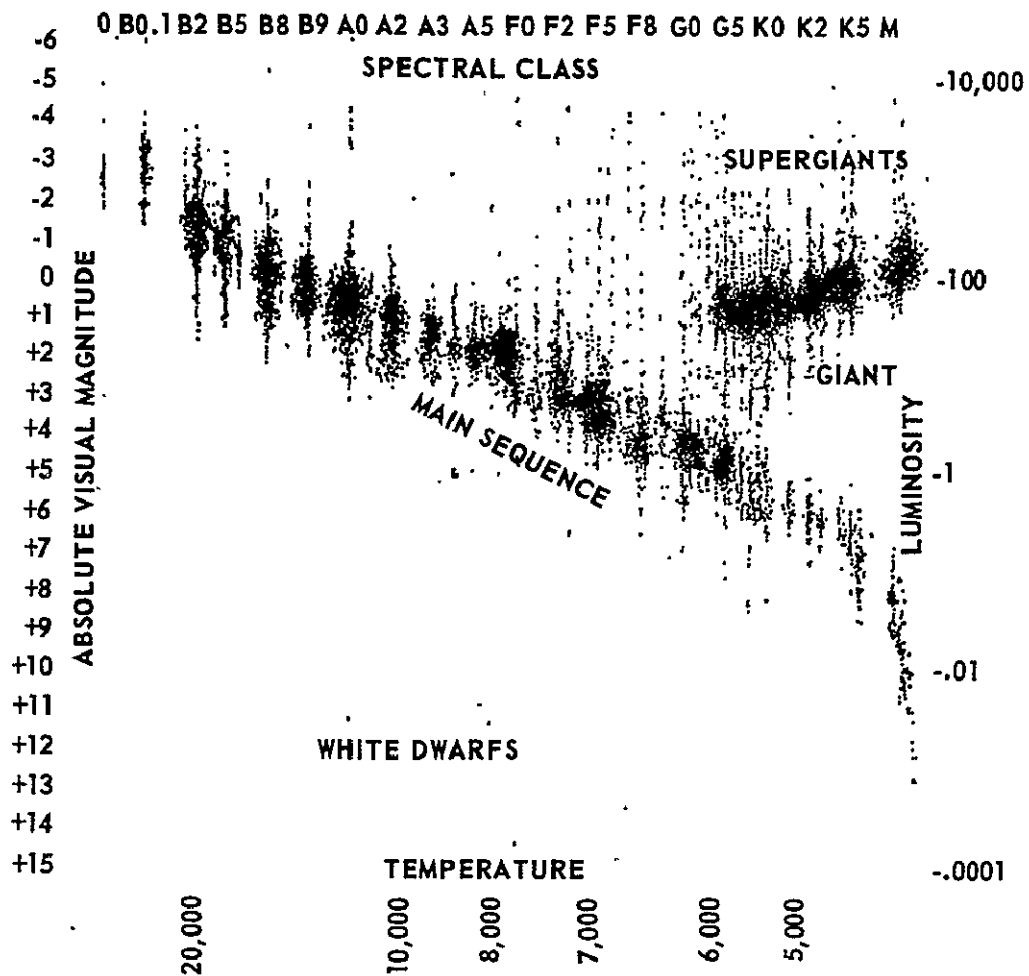


FIGURE B-1. W. GYLLENBERG'S H-R DIAGRAM

large portion of stars is also seen to lie in the upper right of the diagram, the giants. At the top of the diagram resides the supergiants. Finally, in the lower left is the white dwarfs. It is estimated that about 90 percent of the stars, at least in our part of space, are main sequence with about 10 percent of the stars representative of the white dwarf class [1, p. 406]. The giants and supergiants comprise 1 percent or less.

The most widely used system of classifying stars according to their luminosities is that of W. W. Morgan and his associates at the Yerkes Observatory. The stars have been divided into six categories, called luminosity classes, that depend on their luminosities. These classes are as follows [1, p. 406]:

- Ia     Brightest supergiants
- Ib     Less luminous supergiants
- II     Bright giants
- III    Giants
- IV     Subgiants (intermediate between giants and main sequence)
- V     Main Sequence

A small number of stars that may lie below the normal main sequence are called subdwarfs (sd). The white dwarfs are denoted wd. These classes are shown graphically in Figure B-2 [1, p. 407].

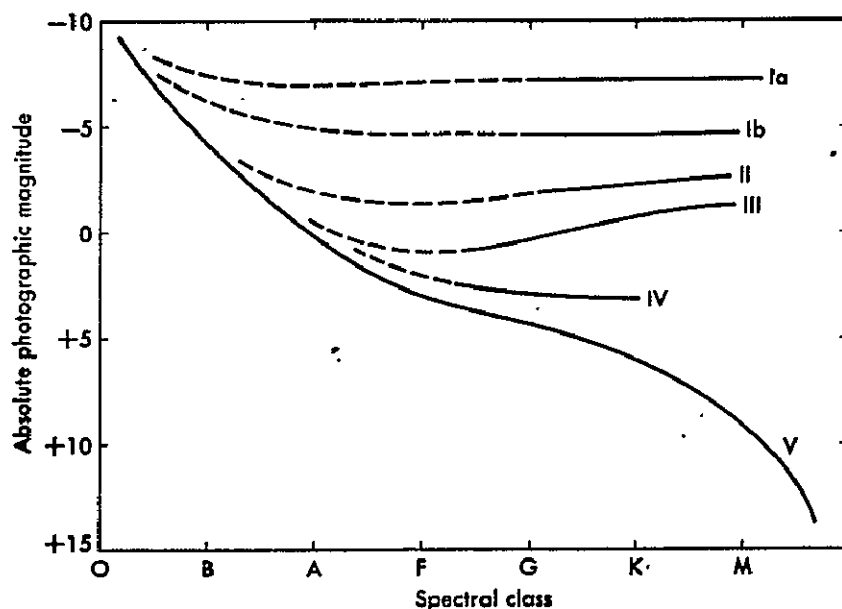


FIGURE B-2. LUMINOSITY CLASSES ON THE H-R DIAGRAM

The theory of stellar structure predicts that a star's position in the chart depends upon its mass, initial chemical composition, and age. From theory we can also deduce the location in the H-R diagram of the main sequence for stars having a given initial composition, as well as how the location of the main sequence changes as nuclear reactions in the stellar interiors alter the

average molecular weight and structure. There is also a tendency for particular kinds of variable stars to occupy specific domains [51]. Obviously, different histories are represented by these aspects of the H-R diagram.

As was stated in the preceding paragraph, stellar structure depends upon mass. Therefore, pictured below is the famous version of the mass-luminosity relation in Figure B-3 [20, p. 260]. "It is found that giants have low average densities, ranging from 0.1 times the sun's average density for a blue giant to 0.00001 for a red giant. Dwarfs (meaning main sequence) are relatively compact; the average throughout the interior ranges from 0.1 to solar value for a hot main-sequence star to 3 times for a red main-sequence object. In white dwarf stars, the densities are exceedingly high" [20, p. 260].

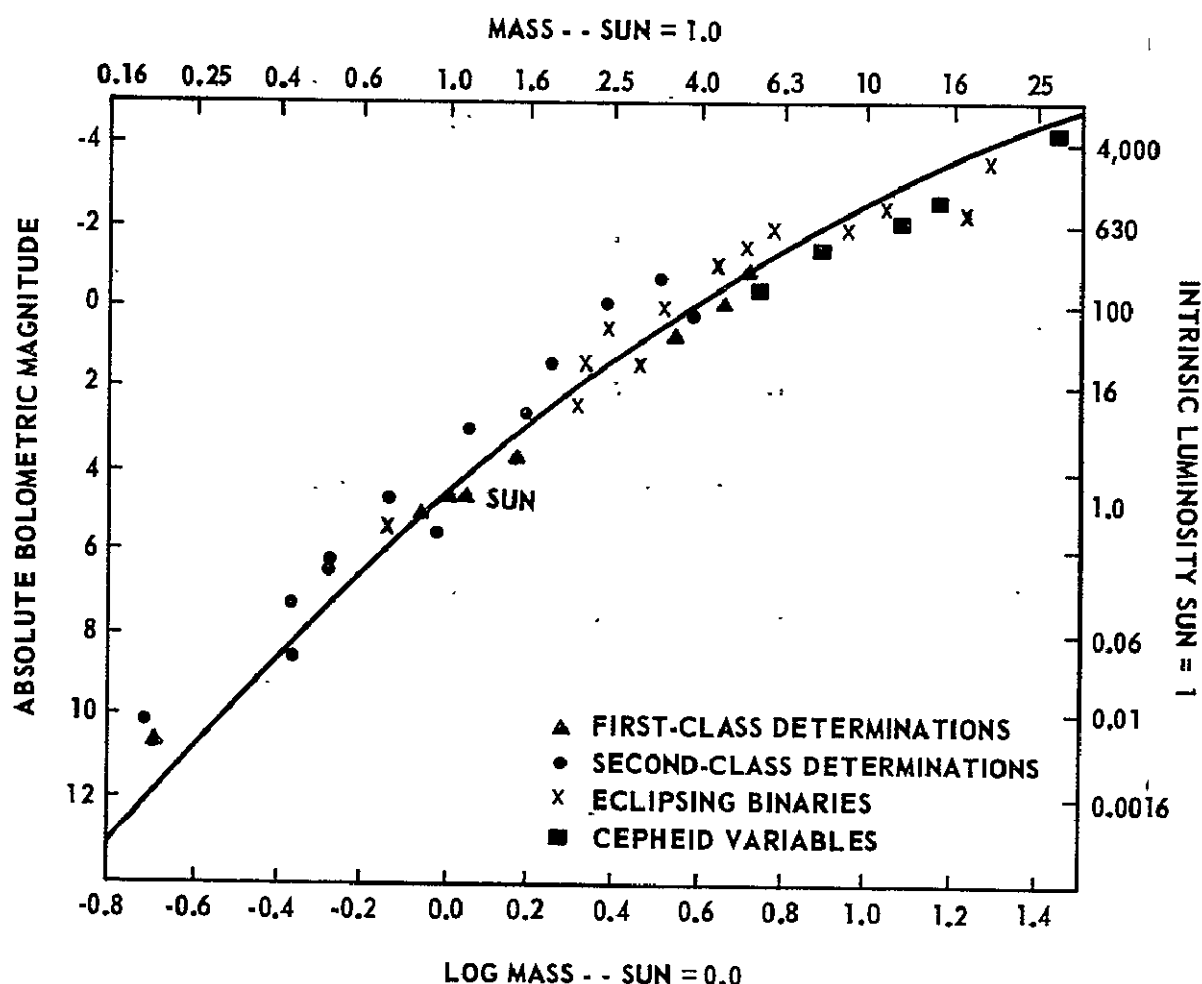


FIGURE B-3. THE MASS LUMINOSITY RELATION

(A famous version of the mass-luminosity relation.

The solid curve represents Eddington's theoretically computed relation.)

"From the empirical mass-luminosity relation, we know that luminosity  $L$  is proportional to a power of the mass  $M$ . This power is slightly greater than two for the high luminosity and low luminosity stars, and slightly less than five for stars of average brightness. Assume that  $L$  is proportional to  $M$ . The total fund of energy a star can expend is directly in proportion to its mass, and the rate of energy expenditure varies directly as the luminosity. Therefore, a star's life-span  $t$  is proportional to  $M/L$ , and thus to  $M/M^4$  or  $1/M^3$ . This means that the more luminous a star is, the more rapidly it burns its nuclear fuel and the shorter is its life" [51, p. 334].

What happens to stars that have left the main sequence? For most clusters there is an obvious region of avoidance between the main sequence and the giant branch. This is the Hertzsprung gap. The younger the cluster, the wider the gap. The red giants or supergiants will be found at just the brightness at which the main sequence stops. Since the brighter stars evolve faster, we have little opportunity to observe such a star during the relatively short interval of its life while it is moving from the main sequence to the red giant region. This explains why few, if any, stars occupy the Hertzsprung gap.

"Before a star becomes a white dwarf, it evolves through a period of instability and occupies a region of the H-R diagram where variable stars are found. The German astrophysicist R. Kippenhalm and his associates find that a massive star in the cepheid variable region will move five times from left to right and back. Figure B-4 is a graph of this phenomenon" [51, p. 336].

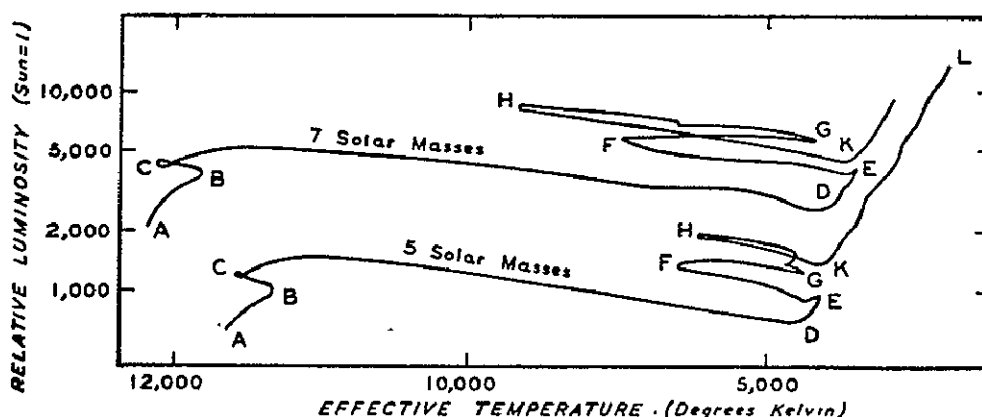


FIGURE B-4. EVOLUTIONARY TRACKS OF TWO CEPHEIDS IN AN H-R DIAGRAM

"As a star of mass seven brightens from A to C, the source of its energy is the conversion of hydrogen to helium, and this step lasts 26 000 000 years. Next, the star cools from C to E for 600 000 years, while its inert core of helium is surrounded by a shell in which hydrogen serves as fuel. At E, however, the core has become hot enough to convert helium into carbon, while hydrogen burning continues in the shell. In this manner, the star evolves from E to F in 5 000 000 years, and from F to G in 1 500 000 more. Evolution from G to H is slow (2 500 000 years) and rapid from H to K (100 000 years); these stages both have an inert core of carbon, surrounded by a layer in which helium is being converted to carbon" [51, p. 336].

The last phase of stellar life seems to be the white dwarf. Theory indicates that a white dwarf cannot be stable if its mass is larger than 1.4 suns. Therefore, it may be probable that the massive stars undergo one or more catastrophes, like supernova or nova explosions. Stars of solar mass or less probably become white dwarfs more quietly [51, p. 336].

Recently [51, p. 336] it has been suggested that possibly white dwarfs can contract further until the hypothetical neutron stars are all that remain. They would have temperatures of perhaps one billion degrees, radii of a few miles, and masses like that of the sun. Their maximum radiation would be in the x-ray region of the spectrum.

On the following pages are two H-R diagrams; the first, being a general schematic type H-R diagram; the second, being an enlarged (and more detailed) version of the central portion of the first diagram. They are Figures B-5 [51, p. 332] and B-6 [51, p. 333], respectively.

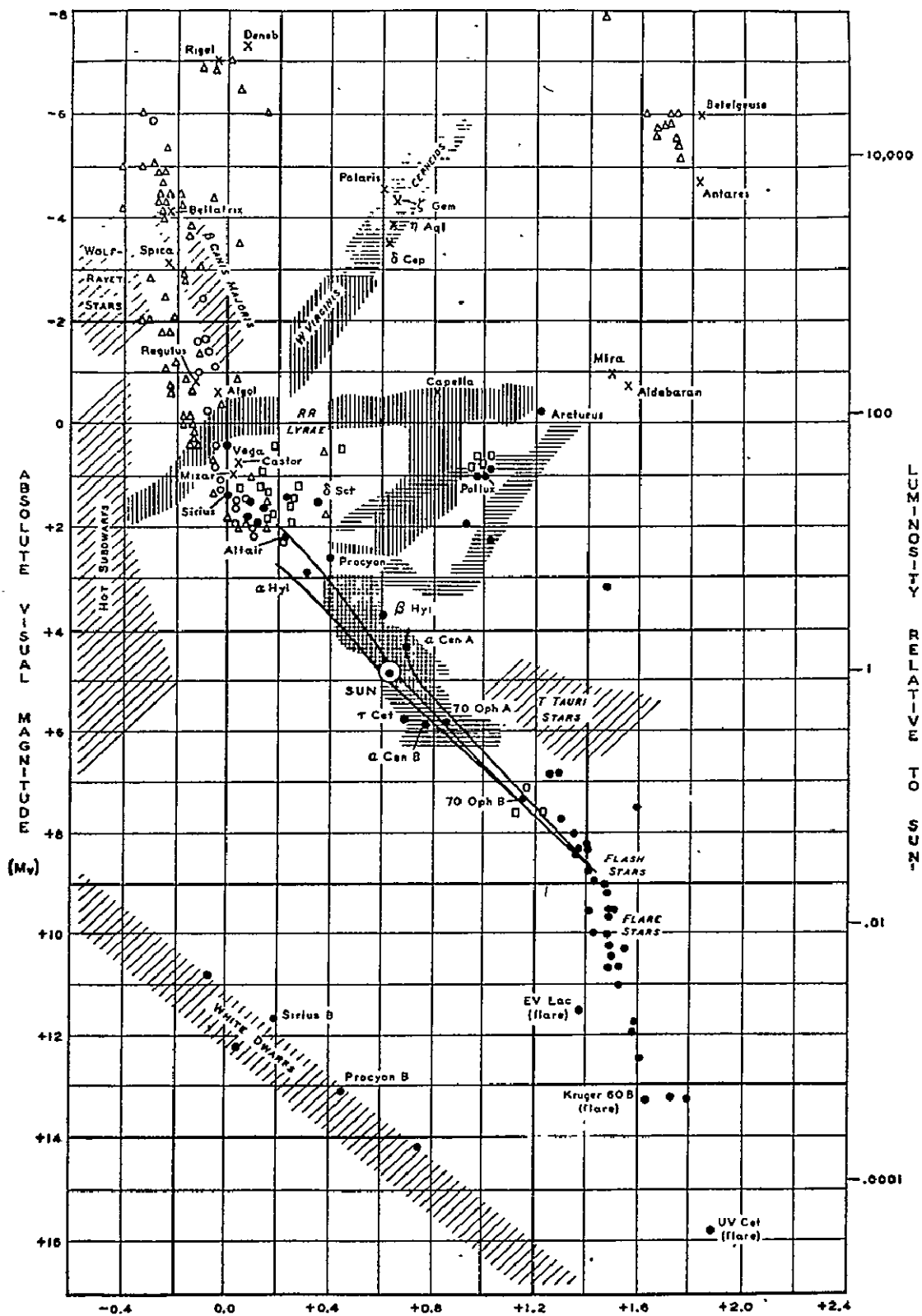


FIGURE B-5. A SCHEMATIC H-R DIAGRAM

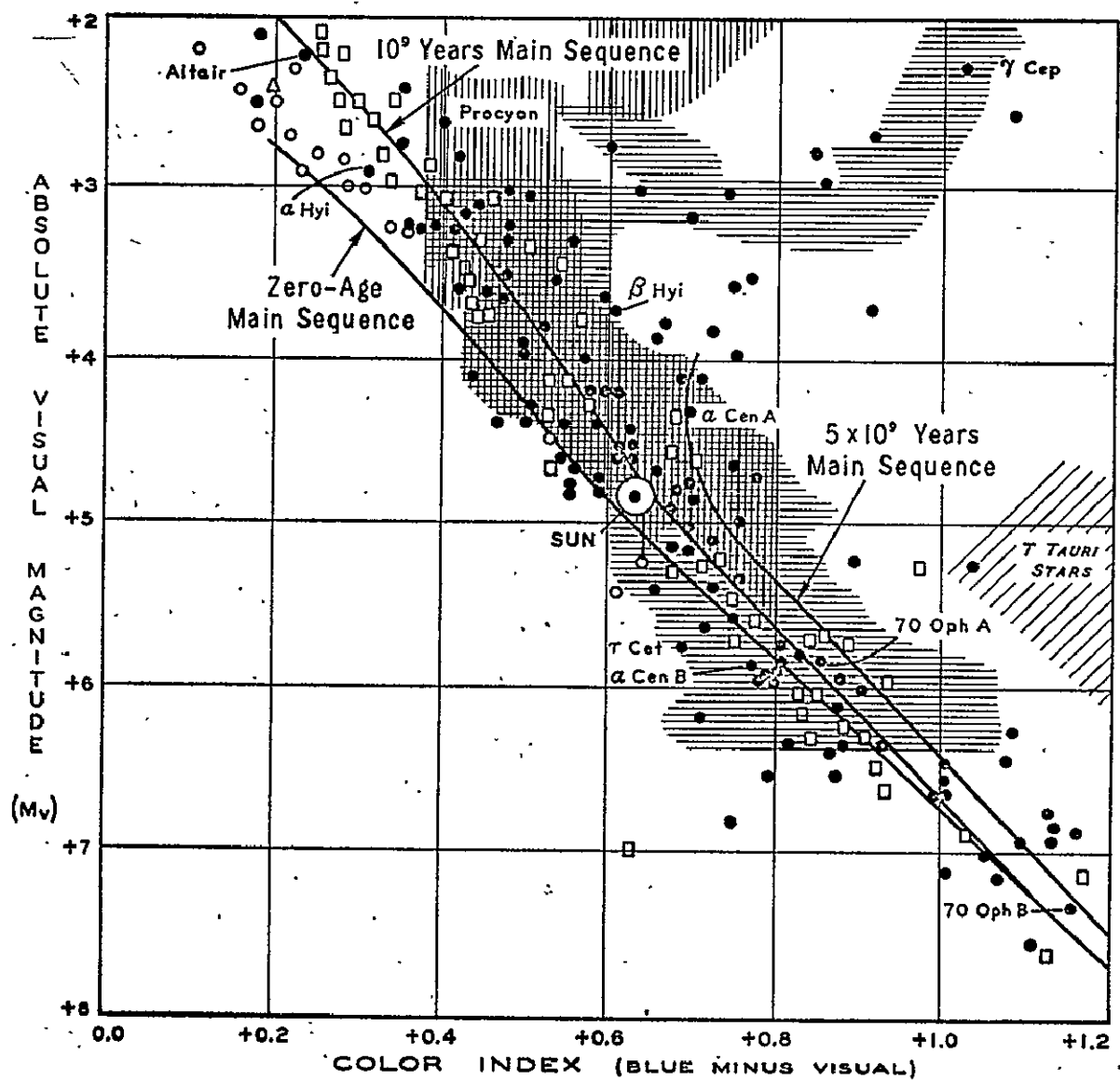


FIGURE B-6. AN ENLARGED H-R DIAGRAM, SOLAR PORTION



## APPENDIX C

### PLANCKIAN RADIATION

[1, pp. 354-356].

A perfect radiator, also called a black body, is an idealized body that completely absorbs all of the electromagnetic energy incident upon it, heats up to a certain temperature, and emits radiation at exactly the same rate as it receives it, and also remains in thermal equilibrium.

"Because of the high opacity of the gases that constitute stars, the light from stars resembles, approximately, that from a perfect radiator. The resemblance is not complete because the temperature increases inward through a stellar photosphere. Moreover, because the opacity of the photosphere to radiation depends on the wavelength of the radiation, the depth within the photosphere from which a given fraction of the emerging light comes depends on the color of that light. However, the spectral energy distribution of a star (the distribution of its light among various wavelengths) usually can be described to a satisfactory approximation by the energy emitted by a perfect radiator at a temperature more or less representative of that of the star's photospheric layers."

"The German physicist Max Planck (1858-1947) found a theoretical interpretation for black-body radiation. He succeeded in this by adopting the hypothesis that light energy is quantized, that is, it occurs only in discrete "packets," or photons, each of energy (in ergs)  $hc/\lambda$ , where  $h = 6.6252 \times 10^{-27}$ ,  $c$  is the velocity of light (cm/sec) and  $\lambda$  is the wavelength (cm). An equation derived by Planck gives the radiant energy emitted per second at any wavelength from a square centimeter of the surface of a black body of any temperature." If  $T$  is the temperature in degrees Kelvin,  $\lambda$ , the wavelength in centimeters, and  $k$ , Boltzmann's constant ( $1.37 \times 10^{-16}$ ),  $E(\lambda, T)$ , the energy in ergs emitted per unit wavelength interval per second per square centimeter and into unit solid angle is

$$E(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}.$$

Figure C-1 shows the energy emitted at different wavelengths for black bodies at several temperatures.

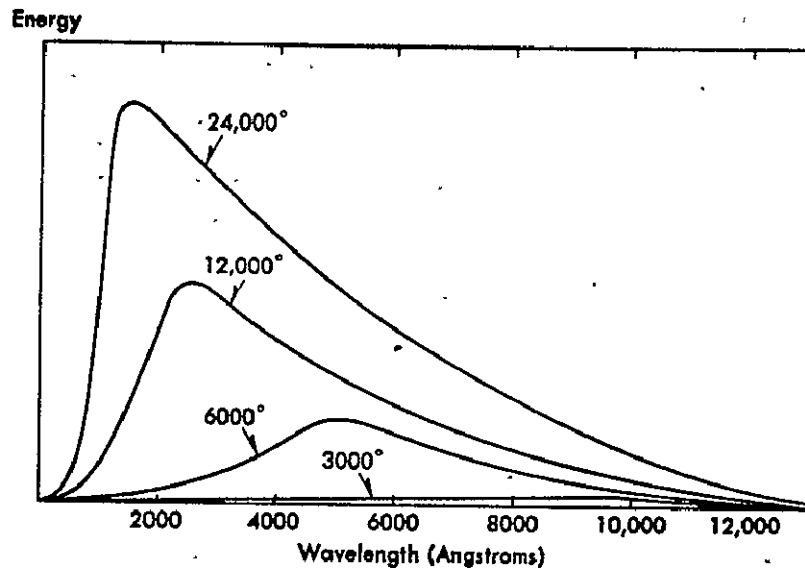


FIGURE C-1. ENERGY EMITTED AT DIFFERENT WAVELENGTHS FOR BLACK BODIES AT SEVERAL TEMPERATURES

The graph can be summarized into three points:

1. A perfect radiator at any temperature emits some radiation at all wavelengths, but not in equal amounts.
2. A hotter black body emits more radiation (per square cm) at all wavelengths than does a cooler black body.
3. A hotter black body emits the largest portion of its energy at shorter wavelengths than a cooler black body does. Hot stars appear blue because most of their energy is at short wavelengths, cool stars appear red because most of their energy is at long wavelengths.

From the graph it is noted that there is a particular wavelength,  $\lambda_{\text{max}}$ , at which a perfect radiator emits its maximum light. The higher the temperature of the body, the shorter  $\lambda_{\text{max}}$ ; in fact,  $\lambda_{\text{max}}$  is inversely proportional to the temperature:

$$\lambda_{\text{max}} = \frac{\text{Const}}{T} \quad (\text{Wien's Law})$$

If the wavelength is measured in centimeters and the temperature in degrees Kelvin, the constant has the value 0.2897.

Wien's law can be derived from Planck's formula by finding the wavelength at which the derivative of Planck's formula equals zero.

If we sum up the contributions from all parts of the spectrum, we obtain the total energy emitted by a black body over all wavelengths; the total energy emitted per second per square centimeter by a black body at a temperature  $T$  is given by the equation known as Stefan's Law:

$$E(T) = \sigma T^4$$

where  $\sigma$ , called Stefan's constant, has the value  $5.672 \times 10^{-5}$ , if  $T$  is in  $^{\circ}\text{K}$ , and  $E(T)$  is in  $\text{ergs}/\text{cm}^2/\text{sec}$ . It can be derived from Planck's law by integrating the Planckian function. Note that a blue, hot star emits more energy per unit area than does a cool, red star.

The energy emitted per unit area of a star (Stefan's law), multiplied by its entire surface area, gives the star's total output of radiant energy, its luminosity. Since the surface area of a sphere of radius  $R$  is  $4\pi R^2$  the luminosity of a star is given by

$$L = 4\pi R^2 \times \sigma T^4$$

This equation can be solved for the radius of the star. The effective temperature is the appropriate kind of temperature to use in the calculation of stellar radii.

By using the above equation as a basis, we can compare an unknown star with a known star (like the sun) and arrive at certain ratios. That is

$$\frac{R_*}{R_{\odot}} = \sqrt{\frac{L_*}{L_{\odot}}} \left( \frac{T_{\odot}}{T_*} \right)^2$$

where  $*$  stands for the star and  $\odot$  stands for the sun.

## APPENDIX D

### INFRARED ASTRONOMICAL OBSERVATIONS

[50, pp. 193-194; pp. 198-202]

"During recent years there has been more and more emphasis upon astronomical measurements in the infrared. To a great extent, this increased emphasis has been stimulated by recent detector developments, which have produced detectors capable of making the difficult long-wavelength observations." W. A. Baum has reviewed the characteristics of photosensitive devices, including infrared detectors, and discussed the spectral responses and sensitivities of most of the available detectors (Annual Review of Astronomy and Astrophysics, Volume 2, p. 165, 1964).

"Among the first observers to make astronomical infrared measurements were Pettit and Nicholson, whose thermoelectric observations included much stellar radiation in the Intermediate Infrared ( $1\ \mu$  to  $4\ \mu$ ) and a small contribution from the Far Infrared ( $>4\ \mu$ ). Until very recently, their data were the observational basis of our knowledge of the bolometric corrections and effective temperatures of stars other than the Sun. About 1950, Fellgett made stellar observations in the Intermediate Infrared, using a lead sulfide cell; Whitford used the same type of detector to make observations of the infrared characteristics of interstellar extinction."

"The year 1963 saw the first publication of photometric data confined entirely to the Far Infrared. Murray and Wildey made stellar and planetary observations through the  $8$  to  $14\ \mu$  atmospheric window and Johnson and Mitchell published the first stellar observations through the  $4.4$  to  $5.5\ \mu$  window. Two years later, the first stellar observations through the  $20\ \mu$  window were published by Johnson, Low, and Steinmetz."

"Ground-based infrared astronomical measures can be made only through the several atmospheric windows. Some infrared spectral regions, such as those around  $1.8\ \mu$ ,  $2.8\ \mu$ ,  $6.5\ \mu$ , and beyond  $22\ \mu$  are closed to ground-based observations because of the very great atmospheric absorption. Observations must be planned around this restriction, but it is now plain that probably 75 percent of the astronomical infrared data that we would like to have can be obtained from good ground-based observing sites."

Observations in both the Intermediate and Far Infrared, bearing upon the problem of interstellar extinction, have been published by Johnson and Borgman, together, and Johnson, alone. "Their interpretation of the long-wavelength data is not compatible with the earlier conclusion, based upon more limited data, that the interstellar extinction law is everywhere the same. Instead, the ratio of total to selective extinction,  $R = A_v / E_{b-v}$ , may be generally about 3.5 with regional excursions to much higher values, mostly (but not exclusively) in the vicinities of very hot stars. Furthermore, the details of the extinction curves in the infrared do not fit well with the computations of van de Hulst."

"Another field of stellar astronomy to which modern infrared measures have made a significant contribution is that of bolometric corrections and effective temperatures; this is the field in which Pettit and Nicholson made their classic contribution 37 years ago." Two analyses of modern infrared photometry in terms of the colors, bolometric corrections, and effective temperatures of stars have been published.

Johnson has defined a multicolor photometric system; this system comprises ten wide-band magnitudes as follows: U (0.36  $\mu$ ), B (0.44  $\mu$ ), V (0.55  $\mu$ ), R (0.70  $\mu$ ), I (0.90  $\mu$ ), J (1.25  $\mu$ ), K (2.2  $\mu$ ), L (3.4  $\mu$ ), M (5.0  $\mu$ ), and N (10.2  $\mu$ ); the effective wavelength of each magnitude is indicated in parenthesis following the letter name of the band.

Almost all supergiants are affected by interstellar reddening. However, it was possible to derive reasonably accurate intrinsic colors for these stars by taking advantage of the facts that "temperature reddening" and intrinsic reddening have different effects upon the several color indices (this was the method used by Kron to derive his supergiant and cepheid intrinsic colors), and that the law of interstellar reddening is not everywhere the same.

The bolometric correction B.C. is defined as  $m_{bol} - V$ , and is the correction to be applied to the V magnitude to obtain the apparent bolometric magnitude. The zero-point has been set so that B.C. = 0.00 for the Sun. A table showing the intrinsic colors, bolometric corrections and effective temperatures is given in Table D-I.

The effective temperature  $T_e$  of a star is defined as the temperature of a black body that produces the same total energy per unit surface area as does the star. Therefore, to compute the effective temperature of a star, a knowledge of both its apparent angular diameter and the total flux of energy received

TABLE D-I. SUPERGIANTS

Sp (I)	$U-V$		$B-V$	$V-R$	$V-I$	$V-J$	$V-K$	$V-L$	$V-M$	$V-N$	B.C.	$T_e$ (°K)
	Ia	Ib										
O9	-1.46	-1.46	-0.31	-0.15	-0.47	-0.73	-0.94	-1.00	—	—	—	—
O9.5	-1.44	-1.44	-0.30	-0.13	-0.45	-0.70	-0.90	-0.96	—	—	—	—
B0	-1.38	-1.37	-0.27	-0.11	-0.38	-0.62	-0.79	-0.84	—	—	-2.29	21000
B0.5	-1.32	-1.30	-0.25	-0.09	-0.32	-0.54	-0.70	-0.74	—	—	-1.99	18500
B1	-1.26	-1.24	-0.22	-0.08	-0.27	-0.47	-0.60	-0.62	—	—	-1.82	16000
B2	-1.15	-1.10	-0.18	-0.05	-0.20	-0.36	-0.45	-0.46	—	—	-1.53	14000
B3	-1.03	-0.98	-0.14	-0.02	-0.14	-0.29	-0.36	-0.36	—	—	-1.27	12800
B5	-0.86	-0.78	-0.10	+0.02	-0.05	-0.20	-0.22	-0.20	—	—	-0.91	11500
B6	-0.80	-0.74	-0.08	+0.02	-0.03	-0.16	-0.16	-0.13	—	—	-0.85	11000
B7	-0.73	-0.67	-0.06	+0.02	-0.01	-0.12	-0.10	-0.06	-0.10	-0.10	-0.75	10500
B8	-0.64	-0.57	-0.03	+0.02	+0.02	-0.08	-0.03	+0.01	-0.03	-0.03	-0.64	10000
B9	-0.58	-0.50	-0.01	+0.02	+0.05	-0.03	+0.03	+0.08	+0.03	+0.03	-0.52	9700
A0	-0.47	-0.40	+0.01	+0.03	+0.08	+0.01	+0.10	+0.15	+0.10	+0.10	-0.38	9400
A1	-0.35	-0.29	+0.03	+0.05	+0.11	+0.06	+0.16	+0.21	+0.16	+0.16	-0.30	9100
A2	-0.22	—	+0.05	+0.07	+0.14	+0.10	+0.22	+0.27	+0.22	+0.22	-0.21	8900
I												
A5	+0.05		+0.11	+0.12	+0.25	+0.23	+0.37	+0.42	+0.37	+0.37	+0.01	8300
F0	+0.45		+0.19	+0.21	+0.41	+0.45	+0.63	+0.68	+0.63	+0.63	+0.14	7500
F2	+0.55		+0.25	+0.26	+0.47	+0.54	+0.75	+0.80	+0.75	+0.75	+0.13	7200
F5	+0.66		+0.37	+0.35	+0.58	+0.67	+0.93	+0.98	+0.93	+0.93	+0.11	6800
F8	+1.01		+0.55	+0.45	+0.72	+0.85	+1.21	+1.27	+1.21	+1.21	+0.08	6150
G0	+1.24		+0.70	+0.51	+0.84	+1.03	+1.44	+1.51	+1.44	+1.44	+0.04	5800
G2	+1.47		+0.85	+0.58	+0.98	+1.20	+1.67	+1.75	+1.67	+1.67	-0.04	5500
G5	+1.83		+1.01	+0.67	+1.11	+1.43	+1.95	+2.04	+1.91	+1.94	-0.13	5100
G8	+2.07		+1.03	+0.69	+1.15	+1.45	+1.99	+2.10	+1.95	+1.97	-0.15	5050
K0	+2.38		+1.12	+0.76	+1.24	+1.58	+2.16	+2.28	+2.09	+2.12	-0.22	4900
K1	+2.60		+1.18	+0.80	+1.32	+1.67	+2.29	+2.42	+2.20	+2.23	-0.28	4700
K2	+2.83		+1.25	+0.85	+1.40	+1.78	+2.44	+2.58	+2.32	+2.36	-0.35	4500
K3	+3.10		+1.38	+0.94	+1.57	+2.00	+2.72	+2.86	+2.57	+2.61	-0.50	4300
K4	+3.35		+1.49	+1.04	+1.75	+2.25	+3.00	+3.15	+2.82	+2.88	-0.68	4100
K5	+3.51		+1.62	+1.20	+2.10	+2.71	+3.70	+3.85	+3.50	+3.57	-1.10	3750
M0	+3.57		+1.63	+1.23	+2.17	+2.82	+3.79	+3.96	+3.59	+3.66*	-1.17	3660
M1	+3.61		+1.63	+1.28	+2.27	+2.90	+3.92	+4.09	+3.72*	+3.79*	-1.26	3600
M2	+3.62		+1.64	+1.34	+2.44	+3.08	+4.11	+4.29	+3.91*	+3.98*	-1.39	3500
M3	+3.60		+1.64	+1.48	+2.79	+3.51	+4.58	+4.77	+4.39*	+4.46*	-1.80	3300
M4	+3.36		+1.64	+1.74	+3.39	+4.26	+5.24	+5.44	+5.10*	+5.13*	-2.44	3100
M5	(+3.00)		+1.62	+2.18	+4.14	+5.04	+6.06	+6.31	+6.00*	+6.00*	-3.23	2950

\* Luminosity classes Ib and II. Probably do not apply to luminosity classes Ia and Iab.

from the star per unit area on the earth is necessary; only for the Sun is the diameter known with sufficient precision that an account effective temperature (5800°K) can be computed. There are only 14 stars for which direct determinations of the effective temperatures can be made. They are shown below in Table D-II.

TABLE D-II. THE EFFECTIVE TEMPERATURE CALIBRATION

		$T_e$ (°K)		Comparison $T_e$ (°K)			
				Kuiper	Harris	Popper	BB
Star	B. C.	(diam.)	(I-L) <sub>BB</sub>	(41)	(51)	(34)	fit
$\mu^1$ Sco	-1.45	19 200	22 500	15 800	27 500	-	-
$\beta$ Per A	-0.30	11 500	11 700	-	-	-	-
$\alpha$ CMa	-0.05	9000	11 700	-	9350	9350	-
$\beta$ Aur	+0.06	10 000	10 000	9800	10 500	10 500	-
$\alpha$ Lyr	-0.08	8950	9000	-	-	-	-
Sun	0.00	5800	5800	5713	5784	-	5800
$\alpha$ Boo	-0.62	4250	4250	4075	4090	-	4200
$\alpha$ Tau	-1.09	3860	3830	3800	3780	-	3600
YY Gem	-1.31	3720	3850	3550	3650	3650	3400
$\beta$ Peg	-1.82	3130	3650	3090	3080	-	3100
$\alpha$ Ori	-1.41	3790	3600	3460	3460	-	3400
$\alpha$ Sco	-1.41	3520	3600	3240	3230	-	3400
$\alpha$ Her	-3.55	3400	3000	-	-	-	2700
$\circ$ Cet (max)	-3.5	2600	3000	2400	2360±	-	-

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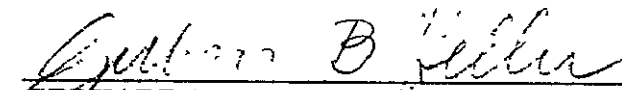
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## A GENERAL SURVEY OF CLASSICAL CEPHEID VARIABLE STARS

By Robert M. Wilson

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Chief, Space Thermophysics Division



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